Volatile Safe-Haven Asset

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Abstract

Despite its high volatility, Bitcoin is alleged to offer diversification benefits through its relatively low correlation with stock markets. However, Bitcoin differs from traditional safe-haven assets; its price is highly sensitive to time-varying correlations and diversification benefits. We find that a decrease (an increase) in correlation between Bitcoin and S&P500 index strongly predicts higher (lower) Bitcoin returns the next day. Following the mean-variance framework, we develop a stylized model of Bitcoin prices utilizing extreme disagreement among Bitcoin investors. When the model is calibrated to Bitcoin's predictability results, it simultaneously explains the lack of predictability in gold and long-term treasuries.

Key words: Safe-Haven Asset, Bitcoin, Time-Varying Correlation, Return Predictability

JEL Classification: G12, G15, D83

Declarations of Interest: None

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1 Introduction

Safe-haven assets receive special attention at the age of uncertainty. Risk-averse investors often rely on a safe-haven asset to diversify their portfolios in hope of hedging recession or inflation risk. Recently, a major cryptocurrency Bitcoin is promoted as a new safe-haven asset by 'digital gold' narrative (Shiller 2020). Consistent with the narrative, numerous studies find that Bitcoin offers a diversification benefit which is possibly time-varying.[1] Traditional investors, such as pension funds and sovereign wealth funds, include Bitcoin in their portfolios for potential diversification benefits.[2]

However, we find that Bitcoin price behaves differently from other traditional safe-haven assets; Bitcoin price is highly sensitive to time-varying correlations and diversification benefits. Our estimation shows that if Bitcoin's time-varying correlation with S&P500 index returns drops (rises) by 0.1, then daily Bitcoin returns on the next day are 1.5% higher (lower) than otherwise would have been. This return predictability is highly significant (t-statistics -3.68) in Bitcoin, but traditional safe-haven assets such as gold and long-term treasuries show no predictability, to the contrary.

The sign of Bitcoin's return predictability looks puzzling at first glance because lower correlation generally implies larger diversification benefits and lower risk premium to compensate. We propose a simple return-predictability mechanism based on trading practices. Consider a *passive* fund manager who holds a portfolio which is a mix of the stock market portfolio and a safe-haven asset. She revises her estimate on the correlation between them at market close and updates optimal portfolio weights on both assets. Then she requests in-house traders or external trading firms to rebalance her portfolio accordingly. However, traders in practice rarely execute orders immediately. Instead, they split and delay orders during the next trading session while hoping for better execution prices without revealing the fund manager's intentions, as shown in the literature theoretically (Kyle 1985, Admati and Pfleiderer 1989) and empirically (Barclay and Warner 1993, Chakravarty 2001).[3] Therefore, an increase (decrease) in correlation today at market close can

[1] For example, see Briere, Oosterlinck, and Szafarz (2015), Dyhrberg (2016a), Dyhrberg (2016b), Bouri, Molnár, Azzi, Roubaud, and Hagfors (2017), Corbet, Meegan, Larkin, Lucey, and Yarovaya (2018), Guesmi, Saadi, Abid, and Ftiti (2019), Shahzad, Bouri, Roubaud, Kristoufek, and Lucey (2019), Akhtaruzzaman, Sensoy, and Corbet (2020) Bouri, Shahzad, Roubaud, Kristoufek, and Lucey (2020), Shahzad, Bouri, Rehman, and Roubaud (2022), Bakry, Rashid, Al-Mohamad, and El-Kanj (2021), and Huang, Duan, and Mishra (2021), among many others.

[2] See news articles at bit.ly/3qxaVmt, bloom.bg/3gl5zG2, bloom.bg/3qCcoIc, on.wsj.com/3htc369, and https://on.wsj.com/37clZgh.

[3] See an article related to this Bitcoin trading behavior at https://yhoo.it/3i9ctPO. Also, see empirical evidence on order imbalance (Chan and Fong 2000, Barber, Odean, and Zhu 2008), and institutional investors (Sias and Starks 1997, Keim and Madhavan 1995).

suppress (boost) the hedging or diversification demands for Bitcoin and thus equilibrium Bitcoin prices on the next day.

Our proposed mechanism can qualitatively explain Bitcoin's return predictability but still leaves some questions unanswered.[4] Why does only Bitcoin display return predictability while other safe-haven assets such as gold and long-term treasuries fail to do so? Is the degree of Bitcoin's predictability quantitatively plausible?

Formalizing the return-predictability mechanism, we develop a stylized model of a safe-haven asset market with active and passive investors under a classical mean-variance framework (Markowitz 1952), without other market frictions such as liquidity or transaction costs. Active investors are overall rational, but they ignore changes in correlation and stick to their own subjective belief (or price target) on a safe-haven asset. By contrast, passive investors avoid price speculation and minimize the total variance of their portfolio. We calibrate the model so that it can replicate Bitcoin's predictability result with empirically plausible parameters, using estimates from the Dynamic Conditional Correlation (DCC) Generalized Autoregressive Conditional Heteroskedasticity (GARCH). Then the calibrated model simultaneously explains the lack of predictability in traditional safe-haven assets such as gold and long-term treasuries.

The key difference between Bitcoin and other safe-haven assets lies in their volatilities. Bitcoin is five times more volatile than S&P500 index in recent years whereas gold is less volatile than S&P500 index. If Bitcoin and gold markets have the same proportion of active investors, then a classical mean-variance framework implies that active investors in Bitcoin markets require a higher risk premium to compensate high volatility than those in gold markets do. That is, the active investors in Bitcoin markets have higher price targets (or expected-return).[5]

We notice that active investors with high price targets should have inelastic asset demands. With high subjective expected returns, the same change in asset prices makes only small percentage changes in subjective expected returns and asset demands. For example, suppose active investors' subjective daily expected return on Bitcoin is 4% since they set their expected Bitcoin price (or price target) on the next day at 4% higher than the current price. If Bitcoin price suddenly drops by 0.2%

[4] See Section 3.5 for alternative explanations.

[5] In fact, this is exactly what is happening in the Bitcoin markets. Unlike traditional safe-haven assets like gold and long-term treasuries, Bitcoin enthusiasts are well known for their extreme optimism, possibly fueled by its exponential price growth in the recent period. The active Bitcoin investors' high price targets imply a strong disagreement on fair asset values among all investors, which explains why Bitcoin has high volatility in the first place.

purely because of passive investors' rebalancing orders, then the new subjective expected return of active investors is about 4.2% ($\approx 4\% + 0.2\%$) which is only 5 percent ($4.2/4.0 - 1 = 0.05$) higher than the previous number 4%. However, if active investors' Bitcoin price target (or expected price) is only 0.1% higher than the current price (equivalently, their subjective expected return of Bitcoin is only 0.1% at the beginning), the expected return would be almost tripled to 0.3% ($\approx 0.1\% + 0.2\%$) by the sudden 0.2% price drop. Therefore, active Bitcoin investors with high price targets have less elastic demands than active gold investors.

If active investors' demand is inelastic, asset price should move more to clear the market when passive investors want to rebalance their Bitcoin position in response to changes in correlation and diversification benefits. In this way, a highly volatile safe-haven asset can show strong return predictability whereas relatively less volatile safe-haven assets should show practically no predictability. Therefore, Bitcoin offers a unique laboratory to test the return-predictability mechanism. Bitcoin is an uncommon asset that attracts both types of investors: active investors with extreme optimism—possibly due to excitement for blockchain technology—and passive investors with strong hedging or diversification demand at the age of uncertainty—possibly due to success of digital gold narrative that diverts attention from slow and expensive Bitcoin transactions (Hinzen, John, and Saleh 2022).[6]

From various angles, we thoroughly investigate Bitcoin's return predictability evidence and find the following. First, the observed return predictability is economically meaningful. Our sample includes 765 observations, R^2 is above 1%, and Bitcoin's daily return volatility is 5%, all of which are comparable to the market return predictability results in the literature with 60 years of monthly data. This return predictability is observable to econometricians but ignorable to traders who face huge intra-day volatilities. Second, other variables can hardly explain the return predictability evidence. The magnitude and the statistical significance of the coefficient estimate remain stable even if other existing predictors or control variables are used in multivariate regressions and machine learning algorithms. Third, out-of-sample R^2 by a robust linear model is 2.3% at daily frequency, which eliminates concern of look-ahead bias and instable coefficients. Lastly, a placebo test and extensive robustness tests validate the return predictability evidence.

To the best of our knowledge, this paper is the first attempt to link *time-varying diversification*

[6]See Hardle, Harvey, and Ruele (2020) for an extensive literature review.

benefits with *return predictability* in old and new safe-haven assets, both theoretically and empirically. Nevertheless, various underlying assumptions of our analysis are closely related to prior studies. First, we assume that different types of investors co-exist in Bitcoin markets. Consistent with our assumption, Ferko, Moin, Onur, and Penick (2021) document that a significant fraction of investors in Bitcoin futures markets also have net long positions in others such as the stock market index futures whereas some investors are highly concentrated in Bitcoin. Second, we use time-varying correlation to track the time-varying diversification benefits as a return predictor. Several researches also estimate time-varying correlations between different asset classes and suggest an effective portfolio strategy to investors in practice (Gao and Nardari 2018).[7]

However, this paper differs from other studies on cryptocurrency markets at least in two aspects. First, we highlight a rational aspect of Bitcoin markets, in particular, asset pricing implications of time-varying hedging or diversification benefits for passive institutional investors. Second, we study Bitcoin's return predictability through its joint dynamics with stock markets. By contrast, many existing studies focus on cryptocurrency markets alone or investigate their irrational side driven by price manipulation or behavioral mistakes by retail investors.[8]

The paper proceeds as follows. Section 2 presents evidence on time-varying correlation and return predictability. Section 3 develops a stylized model of a safe-haven asset market. Section 4 validates Bitcoin's return predictability from various angles. Section 5 discusses remaining concerns and performs extra tests. Section 6 concludes.

2 Motivation: Return Predictability of Safe-Haven Assets

Suppose the correlation between the market portfolio and a safe-haven asset is time-varying. Then the hedging or diversifying benefits of the safe-haven asset must be also time-varying. Therefore, the asset demands and prices of the safe-haven asset should fluctuate accordingly. We first examine this hypothesis with three safe-haven assets: gold, long-term treasuries, and Bitcoin, so-called *digital gold*. Finally, we find strong return predictability of Bitcoin, due to its time-varying correlation with the stock market portfolio.

[7]See Bianchi, Guidolin, and Pedio (2020) for other time-varying aspects of Bitcoin.

[8]See Cheng, De Franco, Jiang, and Lin (2019), Makarov and Schoar (2020), Li, Shin, and Wang (2021), and Griffin and Shams (2020) among many others.

2.1 Time-Varying Correlations and Diversification Benefits

To estimate time-varying correlation between a safe-haven asset and the stock markets, we choose a parsimonious time-series model: DCC(1,1)-GARCH(1,1), Dynamic Conditional Correlation – Generalized Autoregressive Conditional Heteroskedasticity, by Engle (2002).[9] Let a 2×1 vector $\boldsymbol{r}_t = [r_{b,t} \;\; r_{m,t}]^\top$ denote the daily log returns of a safe-haven asset and S&P500 index at time t.

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \mathbf{e}_t \quad \text{where} \quad \mathbf{e}_t \sim N(\mathbf{0}, \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t),$$

where $\boldsymbol{D}_t = diag\{\sigma_{b,t}, \sigma_{m,t}\}$ is a diagonal matrix of the time-varying conditional volatilities, modeled by univariate GARCH(1,1):

$$\sigma^2_{i,t} = \omega_i + \alpha_i e^2_{i,t-1} + \beta_i \sigma^2_{i,t-1} \quad \text{for} \quad i = b, m,$$

while $\boldsymbol{R_t}$ is the conditional correlation matrix of $\mathbf{e}_t$, modeled by DCC(1,1):

$$\boldsymbol{R}_t = \boldsymbol{Q}_t^{*-1} \boldsymbol{Q}_t \boldsymbol{Q}_t^{*-1} \quad \text{where} \quad \boldsymbol{Q}_t = (1 - a - b)\overline{\boldsymbol{Q}} + a \boldsymbol{\epsilon}_{t-1} \boldsymbol{\epsilon}_{t-1}^\top + b \boldsymbol{Q}_{t-1}, \tag{1}$$

where $\overline{\boldsymbol{Q}}$ is the unconditional covariance matrix of $\boldsymbol{\epsilon}_t = \boldsymbol{D}_t^{-1} \mathbf{e}_t \sim N(\mathbf{0}, \boldsymbol{R}_t)$ and $\boldsymbol{Q}_t^*$ is a diagonal matrix whose diagonal elements are the square roots of the diagonal elements of $\boldsymbol{Q}_t$.[10] The model reduces to the Constant Conditional Correlation (CCC) GARCH model of Bollerslev (1990) if $\boldsymbol{R}_t$ is time-invariant $\boldsymbol{R}_t = \overline{\boldsymbol{R}}$ and so $\boldsymbol{Q}_t = \overline{\boldsymbol{Q}}$, which will produce $a \approx 0$ in estimation. To the contrary, correlation is time-varying if parameter a statistically differs from zero.

We apply DCC(1,1)-GARCH(1,1) models to three safe-haven assets: gold, long-term treasuries, and Bitcoin. We use futures prices for gold, spot prices for Bitcoin, and Vanguard Long-Term Treasury Index Fund ETF Shares (VGLT) prices for long-term treasuries.[11] The data ranges from Dec 18, 2017 to Dec 31, 2020, which results in 765 daily observations. We choose the starting date as Dec 18, 2017 because it is when Bitcoin futures markets begin at Chicago Mercantile

[9] Our results are robust to different model choices. See Section A.3.1 in the online appendix for alternative multivariate GARCH models with conditional correlations.

[10] To ensure model validity, parameters a and b satisfy the restrictions $a \geq 0$, $b \geq 0$, $a + b < 1$, and $\boldsymbol{Q}_t$ should be positive definite.

[11] VGLT primarily invests in U.S. treasury bonds and maintains a dollar-weighted average maturity of 10 to 25 years. We download Bitcoin and S&P500 daily closing price data from *coinmarketcap.com* while gold futures and VGLT price data from *investing.com*. See Section 4.1 for our data collection procedures.

Exchange (CME), followed by Chicago Board Options Exchange (CBOE).[12] Since then the nature of uncertainty in Bitcoin changes from idiosyncratic to systematic (Pástor and Veronesi 2009), and Bitcoin trading and price dynamics change significantly, as documented in the literature.[13] Most importantly, investors begin to recognize Bitcoin as a safe-haven because of its new narrative *digital gold*. As a result, more investors participate in Bitcoin markets for hedging and diviersification rather than pure speculations (Ferko, Moin, Onur, and Penick 2021).[14]

The estimated correlations of all three safe-haven assets with S&P500 are all highly time-varying, as shown in Figure 1. We confirm that evidence on time-varying correlation is statistically significant for all three safe-haven assets; see t-statistics of parameter a of DCC(1,1)-GARCH(1,1) in Table 1. Therefore, the hedging or diversification benefits of those safe-haven assets are also time-varying; so are asset demands.

Despite extreme volatilities and relatively high correlation levels, Bitcoin offers a daily diversification benefit for the sample period studied in this paper. Assuming that the DCC(1,1)-GARCH(1,1) estimates are true values, we compute the daily ex-ante variance of the global minimum variance portfolio that includes only S&P500 and Bitcoin. The 83.5% of 765 days in the sample period show reduced ex-ante variance with a positive position in Bitcoin.

2.2 Return Predictability

Table 2 shows how changes in correlation today predict the next day's safe-haven asset returns in a predictive regression:

$$r_{b,t+1} = a_0 + a_1 \Delta\rho_t + \varepsilon_{t+1}, \tag{2}$$

where $\Delta\rho_t$ is a lagged correlation change between a safe-haven asset and S&P500 index. In case of Bitcoin, the estimated slope coefficient a_1 is -0.153, and its Newey-West adjusted t-statistic is -3.7 with R^2 of 1.05%. That is, if correlation drops (rises) by 0.1 today, we can expect about 1.5% higher (lower) Bitcoin returns the next day. This predictability is robust to various specifications: excluding samples after the COVID-19 outbreak, using robust linear models such as LAD (Least

[12] In late 2017, Bitcoin experiences an iconic event; Bitcoin futures markets begin at Chicago Board Options Exchange (CBOE) on December 11 and at Chicago Mercantile Exchange (CME) on December 18, 2017. CBOE delisted Bitcoin futures in 2019 temporarily but CME never did.

[13] See, for example, Augustin, Rubtsov, and Shin 2020, Hardle, Harvey, and Ruele 2020, and Kim, Lee, and Kang 2020

[14] We find that the correlation between Bitcoin and S&P500 is constant before Dec 18, 2017 anyway.

Absolute Deviation) and Rank regressions, transforming the data by Inverse Normal Transformation, or trimming the data. In all alternative specifications, the predictability evidence is stable or even stronger than the baseline case.

We notice that these results are highly reliable and economically meaningful; we have 765 observations, R^2 is above 1%, and Bitcoin's daily return volatility is about 5%, all of which are comparable to the market return predictability results in the literature with 60 years of monthly data.[15] We provide a thorough empirical analysis (Section 4), a placebo test (Section 5.2), and extensive robustness tests (Section 5.3) on this return-predictability evidence.

However, the sign of Bitcoin's return predictability seems puzzling at first glance because lower correlation generally implies larger diversification benefits and lower risk premium to compensate risk. Furthermore, Table 2 shows that changes in correlation fail to predict the next-day returns of traditional safe-haven assets such as gold and long-term treasuries. We argue that Bitcoin's predictability is not just a lucky outcome even after adjusting multiple testing problem; Bitcoin fundamentally differs from traditional safe-haven assets. The model in the next section explains these puzzling empirical patterns.

3 Safe-Haven Asset Returns: A Static Model

Inspired by a well-known asset allocaiton practice, we develop a stylized but parsimonious partial equilibrium model. The model aims to quantify the effect of time-varying correlation on subsequent returns, holding other things (e.g., exotic features of Bitcoin) constant, rather than perform complete valuation of a safe-haven asset.[16]

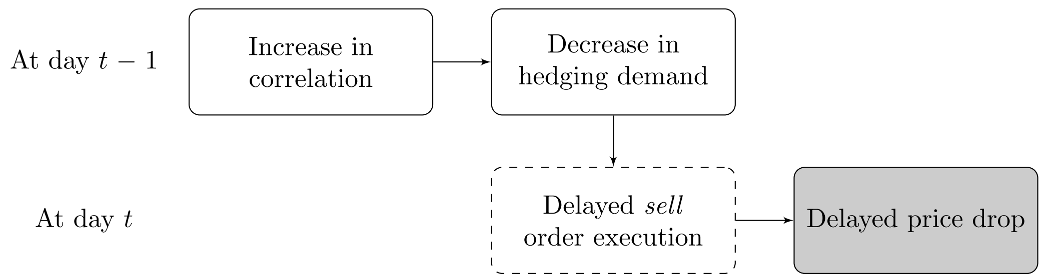
3.1 Return-Predictability Mechanism: A Summary

Prior to a formal model, we present a non-technical summary of return-predictability mechanism: an increase (decrease) in correlations today at market close suppresses (boosts) the hedging or

[15]Bitcoin's high volatility can be also interpreted as a subordinated process: Bitcoin behaves as if the clock for Bitcoin runs fast. This interpretation also supports the use of daily returns data for a short period.

[16]Many non-standard components are used in the recent Bitcoin pricing literature, e.g., gradual adoption, Bitcoin mining cost, payment transaction volume, size of network, halving, and sentiments. See Athey, Parashkevov, Sarukkai, and Xia (2016), Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2020), and Cong, Li, and Wang (2021) among many others. Ignoring these exotic forces in Bitcoin valuation is unrealistic, yet unifying all of them is not only challenging but creating more confusion.

diverfication demands for Bitcoin and Bitcoin prices on the next day.



The above flowchart visualizes the mechanism behind the predictability which is observable to econometricians but ignorable to traders who face huge intra-day volatilities. This one-day time lag in rebalancing is negligible at monthly frequency; therefore, conventional empirical tests of canonical asset pricing models easily overlook this short-term predictability.

This mechanism is closely linked to asset allocation in practice. Suppose a *passive* fund manager holds a portfolio which is a mix of the benchmark portfolio and a safe-haven asset. She revises her estimate on the correlation between them at market close and updates optimal portfolio weights on both assets. Then she requests in-house traders or external trading firms to rebalance her portfolio accordingly. The traders in practice rarely execute orders immediately. Instead, they split and delay orders during the next trading session while hoping for better execution prices without revealing the fund manager's intentions (Kyle 1985, Admati and Pfleiderer 1989).[17] At an aggregate level, such delayed rebalancing demands are highly correlated across passive investors who observe the same public information such as historical prices. Therefore, an increase (decrease) in correlations today at market close suppresses (boosts) the hedging or diverfication demands for Bitcoin and Bitcoin prices on the next day.

The delayed-trading practice is supported by rich empirical evidence. Stealth trading is prevalent in the markets (Barclay and Warner 1993, Chakravarty 2001), order imbalance predicts daily returns (Chan and Fong 2000, Barber, Odean, and Zhu 2008), and institutional investors' behaviors create return autocorrelation at daily level (Sias and Starks 1997, Keim and Madhavan 1995). However, our model and empirical finding differ from these studies which find transitory price impacts by heterogeneous traders. By contrast, the price impact in our model and data is practically

[17]See an article related to this Bitcoin trading behavior at https://yhoo.it/3i9ctPO.

permanent because the correlation dynamics is highly persistent (close to a random walk); the parameter b in DCC-GARCH model is estimated as 0.948 in Table 1 which roughly measures the serial-correlation of time-varying correlation.

We emphasize that this mechanism does not necessarily require investors' daily rebalancing. For example, they maybe rebalance once a month but on different days so that there are some investors who rebalance their portfolios on any given day.[18] Furthermore, even if this delayed rebalancing occurs only once in a few days, return predictability can appear as an averaged daily effect in empirical tests with a dataset of multiple days. We formalize this return-predictability mechanism in a partial equilibrium pricing model in the following sections.

3.2 Asset Demands from Two Types of Investors

The mechanism in the previous section can qualitatively explain Bitcoin's return predictability but still leaves some questions unanswered. Why does only Bitcoin display return predictability, but neither gold not long-term treasuries? Is Bitcoin's predictability quantitatively plausible? To answer these questions, we develop a model of a safe-haven asset as follows.

Consider two types of investors in the market of a safe-haven asset: *active* and *passive* investors.[19] Then we model their asset demands in the same mean-variance framework but with different input parameters for portfolio optimization. They mix a safe-haven asset with their existing well-diversified risky portfolio such as stock market portfolios, say, an S&P500 index fund, to maximize Sharpe ratio.[20] We let $\mu_{b,t}$ and $\mu_{m,t}$ denote the conditional subjective risk premia of a safe-haven asset and the stock market, respectively. Similarly, σ_b and σ_m denote the volatilities of the two assets, respectively. Then the investors' ex-ante optimal weight on a safe-haven asset in the new tangency portfolio is given by

$$w_b = \frac{\sigma_m^2 \mu_{b,t} - \rho_t \sigma_b \sigma_m \mu_{m,t}}{\sigma_m^2 \mu_{b,t} - \rho_t \sigma_b \sigma_m \mu_{m,t} + \sigma_b^2 \mu_{m,t} - \rho_t \sigma_b \sigma_m \mu_{b,t}} = \frac{\mu_t^* - \rho_t \sigma^*}{(\mu_t^* - \rho_t \sigma^*) + (\sigma^* - \rho_t \mu_t^*)\sigma^*}, \tag{3}$$

where ρ_t is the correlation coefficient between a safe-haven asset and the stock market, and where

[18] This alternative assumption only results in a different interpretation on the model: now the model is about the investors who participate in trading on a given day rather than all investors.

[19] The distinction between two types of investors are not physical. One investor can have both active and passive aspects, and then we can interpret her asset demand as a sum of the demands from two different types of investors.

[20] Even if investors in the real world do not maximize Sharpe ratio, we can still rely on the mean-variance framework to explain their investment decisions simply by adjusting input parameters for portfolio optimization accordingly.

$\mu_t^* = \mu_{b,t}/\mu_{m,t}$ and $\sigma^* = \sigma_b/\sigma_m$ are the safe-haven asset's risk premium ratio and volatility ratio relative to the stock market, respectively.[21]

Both types of investors agree on σ^* but agree to disagree on μ_t^* and ρ_t. In particular, the passive investor believes $\mu_t^* = 1$. In other words, the passive investor is so defensive that her optimal portfolio is the global minimum variance portfolio which is implied by setting $\mu_t^* = 1$.[22] The passive investor believes that predicting asset prices is a difficult task and only adds unnecessary noises to the portfolio optimization. Therefore, she would set $\mu_t^* = 1$ regardless of current price levels. Instead, she focuses on tracking time-varying correlation ρ_t rather than the risk premium. By contrast, the active investor cares less about tracking time-varying hedging or diversification benefits. She is generally more optimistic and seeks potential price appreciations. Therefore, she directs all her limited attention and resources to learn about risk premium μ_t^* and future asset prices. Both types of investors stick to their own belief and optimization rule regardless of the other investor's belief and behavior.

Therefore, the optimal weights on a safe-haven asset in both types of investors' portfolios are determined by Equation (3) with their own input parameter values, respectively. For the passive investor, the optimal weight on a safe-haven asset is $w_{b,t}^{(p)} \equiv w_b(\overline{\mu}^* = 1, \sigma^*, \rho_{t-1})$ where ρ_{t-1} is the correlation estimate from the previous day at market close. This timing difference is due to the passive investors' delayed order execution from the mechanism in Section 3.1. For the active investor, the optimal weight on a safe-haven asset is $w_{b,t}^{(a)} \equiv w_b(\mu_t^*, \sigma^*, \overline{\rho})$ where $\overline{\rho}$ is the active investor's conservative correlation estimate. Note asset prices at day t does not affect passive investors' asset demand, in terms of portfolio weight $w_{b,t}^{(p)}$, which is already set on the previous day at market close. By contrast, a safe-haven asset price $P_{b,t}$ at day t is directly linked to active investors' risk premium estimate μ_t^* and asset demand because we model μ_t^* as a function of the current price $P_{b,t}$ and the active investor's expected future price $E[P_{b,t+1}|\mathcal{F}_t^{(a)}]$ as follows.

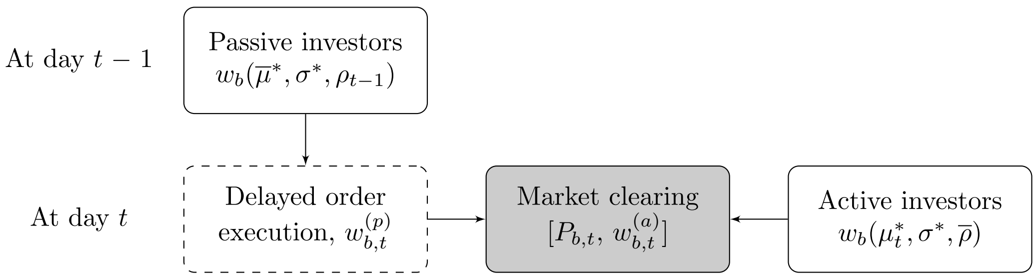
$$\mu_t^* = \frac{\mu_{b,t}}{\mu_{m,t}} = \left(\frac{E[P_{b,t+1}|\mathcal{F}_t^{(a)}]}{P_{b,t}} - R_{f,t} \right) \Big/ \mu_{m,t}, \tag{4}$$

where $\mathcal{F}_t^{(a)}$ is an information set of the active investor at day t and $R_{f,t}$ is a risk-free rate in terms

[21] Refer to any college level textbook on investments for (3).

[22] The condition $\mu_t^* = 1$ implies that the risk premium of the portfolio is invariant with any portfolio weights. Therefore, maximizing Sharpe ratio is equivalent to minimizing variance.

of gross returns. Therefore, the equilibrium price $P_{b,t}$ of a safe-haven asset and the active investors' asset demand, in terms of portfolio weight $w_{b,t}^{(a)}$, will be endogenously determined when the market clears. The following flowchart visualizes how a safe-have asset's market clearing occurs by the two types of investors.



3.3 A Static Equilibrium of a Safe-Haven Asset Market

First, we let $Q_{b,t-1}^{(a)}$ and $Q_{b,t-1}^{(p)}$ denote the quantity of a safe-haven asset held by active and passive investors at day $t-1$, respectively. Then we normalize both the total quantity and price of a safe-haven asset at day $t-1$ to unity, i.e., $P_{b,t-1} = 1$ and $Q_{b,t-1} = Q_{b,t-1}^{(a)} + Q_{b,t-1}^{(p)} = 1$, respectively. Also, we have $Q_{b,t} = Q_{b,t}^{(a)} + Q_{b,t}^{(p)} = 1$ assuming the quantity of a safe-haven asset is invariant overnight. At equilibrium, the price $P_{b,t}$ of a safe-haven asset clears the market:

$$1 = Q_{b,t} = Q_{b,t}^{(a)} + Q_{b,t}^{(p)} = \frac{w_{b,t}^{(a)} A_t^{(a)}}{P_{b,t}} + \frac{w_{b,t}^{(p)} A_t^{(p)}}{P_{b,t}}, \tag{5}$$

where $A_t^{(a)}$ is the dollar value of the active investor's portfolio at day t while $A_t^{(p)}$ is that of the passive investor's portfolio at day $t-1$. Next, the dollar value of the active investor' portfolio evolves as follows.

$$\begin{aligned} A_t^{(a)} &= A_{t-1}^{(a)} \left(w_{b,t-1}^{(a)} \frac{P_{b,t}}{P_{b,t-1}} + (1 - w_{b,t-1}^{(a)}) R_{m,t} \right), \\ &= \frac{P_{b,t-1} Q_{b,t-1}^{(a)}}{w_{b,t-1}^{(a)}} \left(w_{b,t-1}^{(a)} \frac{P_{b,t}}{P_{b,t-1}} + (1 - w_{b,t-1}^{(a)}) R_{m,t} \right), \\ &= Q_{b,t-1}^{(a)} \left(P_{b,t} + \frac{1 - w_{b,t-1}^{(a)}}{w_{b,t-1}^{(a)}} R_{m,t} \right), \end{aligned} \tag{6}$$

where $P_{b,t-1} = 1$ and $R_{m,t}$ is daily gross return of the stock market. Similarly, we express the dollar value of the passive investor' portfolio as follows.

$$A_t^{(p)} = Q_{b,t-1}^{(p)} \left(P_{b,t} + \frac{1 - w_{b,t-1}^{(p)}}{w_{b,t-1}^{(p)}} R_{m,t} \right), \tag{7}$$

Finally, by plugging Equation (6) and (7) into (5), we rewrite the market clearing condition (5) as follows.

$$\begin{aligned} P_{b,t} &= w_{b,t}^{(a)} A_t^{(a)} + w_{b,t}^{(p)} A_t^{(p)} \\ &= w_{b,t}^{(a)} Q_{b,t-1}^{(a)} \left(P_{b,t} + \frac{1 - w_{b,t-1}^{(a)}}{w_{b,t-1}^{(a)}} R_{m,t} \right) + w_{b,t}^{(p)} (1 - Q_{b,t-1}^{(a)}) \left(P_{b,t} + \frac{1 - w_{b,t-1}^{(p)}}{w_{b,t-1}^{(p)}} R_{m,t} \right), \end{aligned} \tag{8}$$

where $w_{b,t}^{(a)}$ is also a function of $P_{b,t}$ because $w_{b,t}^{(a)}$ depends on μ_t^* in Equation (4). Then we numerically find the equilibrium safe-haven asset price $P_{b,t}$, given other variables as calibrated in the following.[23]

Calibration of Variables Two unconditional estimates $(\overline{\rho}, \sigma^*)$ are fixed at their medians of (ρ_t, σ_t^*), from the multivariate GARCH estimation, respectively. Next, to shut down other channels, we set $\rho_{t-2} = \overline{\rho}$, $R_{m,t} = 1$, $R_{f,t-1} = R_{f,t} = 1$, $\mu_{m,t} = \mu_{m,t-1} = 0.06/252$, and $E[P_{b,t+1}|\mathcal{F}_t^{(a)}] = E[P_{b,t}|\mathcal{F}_{t-1}^{(a)}]$. Then, we compute how a safe-haven asset price changes with respect to changes in correlation (from ρ_{t-2} to ρ_{t-1}) for different values of two free parameters $w_{b,t-1}^{(a)}$ and $Q_{b,t-1}^{(a)}$. To calculate $E[P_{b,t}|\mathcal{F}_{t-1}^{(a)}]$, we first back out μ_{t-1}^* from Equation (3), given the values of free parameter $w_{b,t-1}^{(a)}$, and then we compute $E[P_{b,t}|\mathcal{F}_{t-1}^{(a)}]$ from Equation (4).

3.4 Model Implications on Return Predictability

This model is about an equilibrium of a safe-haven asset market where only a subset of investors participate in the market. Therefore, the model considers stock markets as exogenous environments and therefore fundamentally differs from canonical asset pricing models such as CAPM. However, the model is also free from any market frictions such as liquidity or transaction costs that can result in return reversals, except for the passive investor's delayed order execution.

[23] Alternatively, we can plug the $w_{b,t}^{(a)}$ expression into Equation (8) and solve a polynomial equation by brute force.

3.4.1 Model-Implied Coefficient in Predictive Regressions

Table 3 shows the model-implied coefficient a_1 in predictive regressions of a safe-haven asset's log returns $r_{b,t}$ on $(\rho_{t-1} - \rho_{t-2})$, i.e., changes in correlation between a safe-haven asset and the stock market returns:

$$r_{b,t} = a_0 + a_1(\rho_{t-1} - \rho_{t-2}) + \varepsilon_t \quad \text{where} \quad a_1 = \frac{\partial \log P_{b,t}}{\partial \rho_{t-1}}\Big|_{\substack{R_{m,t}=1 \\ \rho_{t-1}=\rho_{t-2}=\overline{\rho}}}, \tag{9}$$

where we deviate ρ_{t-1} from $\overline{\rho} = \rho_{t-2}$ and also $P_{b,t}$ from $P_{b,t-1} = 1$ holding others constant. In Table 3, we compute a_1 using a numerical derivative for each combination of $w^{(a)}_{b,t-1}$ and $Q^{(a)}_{b,t-1}$ ranging from 0.1 to 0.9. The model-implied coefficient a_1 is negative regardless of $w^{(a)}_{b,t-1}$ and $Q^{(a)}_{b,t-1}$, consistent with the mechanism explained in Section 3.1. This calibration exercise confirms whether the model can replicate the coefficient a_1 similar to its empirical estimate. The point estimate on a_1 of the regression using our data is -0.153 for Bitcoin where its 95% confidence interval implied by Newey-West standard errors is $(-0.236, -0.072)$ from Table 2. Therefore, our model can easily produce the empirical estimate on a_1 without extreme parameter values. For example, calibrating $w^{(a)}_{b,t-1} = 0.6$ and $Q^{(a)}_{b,t-1} = Q^{(a)}_{b,t-1}/Q_{b,t-1} = 0.3$ produces $a_1 = -0.154$.

Figure 3 visualizes Table 3 using a finer grid of $w^{(a)}_{b,t-1}$ and $Q^{(a)}_{b,t-1}$. The observed coefficient $a_1 = -0.153$ is about 25.4$^{\text{th}}$ percentile in the plot. The lower left triangular area is where the model-implied coefficient is smaller than -0.153. The patterns in coefficients are straightforward. Given $w^{(a)}_{b,t-1}$, predictability (or magnitude of the coefficient) decreases with the active investor's share in Bitcoin, $Q^{(a)}_{b,t-1}$. High $Q^{(a)}_{b,t-1}$ means that passive investors are only minor players in Bitcoin markets. Therefore, changes in their hedging demands barely affect Bitcoin prices.

On the other hand, Given $Q^{(a)}_{b,t-1}$, predictability increases with active investor's portfolio concentration in Bitcoin, $w^{(a)}_{b,t-1}$. High $w^{(a)}_{b,t-1}$ means that the active investor sees relatively high expected returns on Bitcoin; that is, she has a relatively more optimistic belief. Then the active investor's Bitcoin demand becomes less elastic such that prices should move more to clear the market. For example, assume $\rho_t = 0$ in the optimal portfolio weight in Equation (3). Then we have a simple expression $w_b = \mu^*_t/[\mu^*_t + (\sigma^*)^2]$ for the optimal weight.

Now suppose the risk-free rate is zero and the passive investor wants to raise her position on Bitcoin, $w^{(p)}_{b,t}$, because of declining correlation. Also, imagine that the active investor's risk premium on Bitcoin is 0.4% per day at the beginning since she sets her expected Bitcoin price (or price targets) on the next day at 0.4% higher than the current price. If Bitcoin price suddenly drops by 0.2% purely because of passive investors' rebalancing sell orders, new subjective expected return of active investors becomes about 0.6% ($\approx 0.4\% + 0.2\%$) which is 50 percent ($0.6/0.4 - 1 = 0.5$) higher than the previous number 0.4%. Therefore, the active investor is now willing to absorb passive investor's sell order even with a small price movement because her demand $w_{b,t}^{(p)} = \mu_t^* / [\mu_t^* + (\sigma^*)^2]$ greatly changes. By contrast, if the active investor's Bitcoin price target (or expected price) is only 2% higher than the current price (equivalently, her subjective expected return of Bitcoin is 2% at the beginning), then the expected return becomes about 2.2% ($\approx 2\% + 0.2\%$) which is only 10 percent higher than the previous number 2%. In this case, the active investor might not absorb all of passive investor's sell orders unless the price drops further. Such a behavior of the active investor leads to relatively stronger predictability when $w_{b,t-1}^{(a)}$ is high, or equivalently, when the active investor has a relatively more optimistic belief.

3.4.2 Volatility and Return Predictability

This intuition also explains why a highly volatile safe-haven asset can show stronger return predictability. To have the same level of $w_{b,t-1}^{(a)}$ even with higher volatility, the active investor must have a highly optimistic belief on the risk premium of a safe-haven asset so that strong predictability can be observed in Bitcoin. In fact, this is exactly what happens in the Bitcoin markets. Unlike traditional safe-haven assets like gold and long-term treasuries, Bitcoin enthusiasts are well known for their extreme optimism, possibly fueled by its exponential price growth in the recent period. However, the caveat is that high volatility alone does not guarantee predictability. Consider a small illiquid stock with lottery-like payoffs or a meme stock loved by retail investors. Even if such assets are highly volatile, no hedging or diversification demands exist for them. That is, no predictability will appear because $Q_{b,t-1}^{(a)} \approx 1$ in Table 3.

Figure 4 visualizes 25.4^{th} percentile of the model-implied coefficient a_1 calculated with different levels of correlation $\overline{\rho} = -0.5, 0, +0.5$ and σ^* between zero and five. Then we locate the model-implied coefficients for Bitcoin (brown), gold (red), and long-term treasuries (light blue) as diamond marks, using their DCC(1,1)-GARCH(1,1) estimates on $\overline{\rho}$ and σ^*, respectively. The model implied coefficients for gold and long-term treasuries are practically zero, which is consistent with the empirical results in Table 2. Therefore, this plot implies that the pricing model in this section can quantitatively explain the lack of predictability in gold and long-term treasuries when the model is tuned to match Bitcoin's predictability result with empirically plausible parameters.[24] Table 4 also compares the model-implied coefficient a_1 in Equation (9) with the estimated coefficient a_1 in Equation (2) for Bitcoin, gold, and long-term treasuries. As expected, the model can explain the estimated coefficient a_1 within a reasonable range for Bitcoin, but not for the other safe-haven assets.

Therefore, Bitcoin offers a unique laboratory to test the prosposed mechanism in Section 3.1 and the model in this section. Bitcoin is a rare asset that attracts both active investors with extreme optimism (possibly due to excitement for blockchain technology) and passive investors with strong hedging or diversification demand (possibly due to success of digital gold narrative in uncertain economic times). Finally, with the time-varying correlation and delayed trading practice, we observe predictability in Bitcoin. See Section 5.1 for a further discussion on how to calibrate free parameters.

3.5 Alternative explanations

Although the mechanism in Section 3.1 sounds plausible, we also investigate other possibilities before we formalize this hypothesis in a model. To the best of knowledge, we find that the following two categories can cover all alternative explanations.

Alternative explanation #1: time-varying risk premium Correlations between asset returns are key elements in canonical asset pricing models. The time-varying correlation can imply a time-varying Capital Asset Pricing Model (CAPM) beta in a Conditional CAPM or a state variable in an Intertemporal CAPM. Therefore, the traditional asset pricing models suggest that time-varying CAPM betas or correlation levels should explain the time-variation of Bitcoin risk premium and predict subsequent Bitcoin returns. However, we find that that these variables fail to predict subsequent Bitcoin returns.[25] Furthermore, even if we attribute the lack of predictability to data or econometric issues of daily frequency, the sign of predictability (by changes in correla-

[24] Similarly, we explains the lack of predictability by correlation levels in the online appendix A.2.

[25] Time-varying CAPM beta estimates of Bitcoin are constructed from DCC-GARCH estimates on conditional correlation and volatilities.

tion) contradicts what canonical models suggest. High correlation offers only a small diversification benefit; therefore, the risk premium should be high enough to compensate it. However, we observe lower returns right after an increase in correlation, to the contrary.[26]

Alternative explanation #2: time-varying sentiments Another possibility is that changes in correlation might be a proxy for other variables such as investors' sentiment level. Although this type of conjecture is hard to prove or disprove without an accurate measure of sentiments, we find that this conjecture is highly unlikely. First, Section 4.3 and 5.3 show that the predictability results are unaffected by various sentiment proxies in predictive regressions and machine learning approaches, respectively. Second, if there is a common sentiment for Bitcoin and stock markets, we should observe the opposite Bitcoin price movements the next day in the following two cases: (1) when both assets go up because of high sentiments, vs. (2) when they go down because of low sentiments. However, both cases imply an increase in correlation, which results in lower subsequent Bitcoin returns in the data. Therefore, a common sentiment rather rules out, not explain, return predictability by changes in correlation. Third, variables like common sentiments, excessive liquidity, or degree of disagreement are highly persistent at daily frequency, yet changes in correlation are barely serially-correlated, by construction.

4 Main Results: Bitcoin Return Predictability at Full Scale

Inspired by the model in the previous section, we introduce a more economically sensible predictor for Bitcoin returns:

$$\Delta w_{b,(t-1):t}^{(cor)} = w_{b,t}^{(p)} - w_{b,t-1}^{(p)},$$

where $w_{b,t}^{(p)} \equiv w_b(\overline{\mu}^* = 1, \sigma^*, \rho_{t-1})$ is the passive investor's optimal weight on a safe-haven asset on day t and ρ_{t-1} is the correlation estimate on day $t-1$ at market close from the DCC-GARCH model estimation in Table 1. Following the calibration in the previous section, we set σ^* at the median of σ_t^* from the DCC-GARCH estimation. The resulting predictor $\Delta w_{b,(t-1):t}^{(cor)}$ is a change in optimal portfolio weight due to time-varying correlation. Holding the passive investor's portfolio value equal, this predictor is proportional to a dollar amount of rebalancing order on Bitcoin. Therefore,

[26] A level of correlation tends to be high when changes in correlation is high because $cov(\rho_t, \Delta\rho_{(t-1):t}) = cov(\rho_t, \rho_t - \rho_{t-1}) = var(\rho_t) - cov(\rho_t, \rho_{t-1}) = var(\rho_t)(1 - cor(\rho_t, \rho_{t-1})) > 0$.

this new predictor is economically more sensible although it is highly correlated with changes in correlation ($\Delta\rho_t$). Using this refined predictor $\Delta w_{b,(t-1):t}^{(cor)}$, this section thoroughly investigates Bitcoin's return-predictability: Granger causality tests, in-sample analysis with other competing predictors, out-of-sample predictions, and other robustness tests. All results confirm that the Bitcoin return predictability truly exists when the correlation is time-varying.[27]

4.1 Data

We download Bitcoin and S&P500 daily closing price data from *coinmarketcap.com* and *investing.com*, respectively.[28] Then, we combine multi-day Bitcoin returns to one-period return whenever the US stock market is not traded. The data ranges from Dec 18, 2017 to Dec 31, 2020, which results in 765 daily observations. We call this sample period as *post-futures* in that Dec 18, 2017 is when Bitcoin futures markets begin at Chicago Mercantile Exchange (CME), followed by Chicago Board Options Exchange (CBOE). SKEW, VIX, and S&P500 trading volume data are acquired from *investing.com*. In addition, we collect other variables that are known to predict Bitcoin returns in literature (e.g., Liu and Tsyvinski 2021): USD index returns and gold returns from *investing.com*, daily treasury yield rates from the US Department of Treasury, blockchain-related attributes including total hashrates, global block difficulty, unique address count, total Bitcoin quantity, unique transactions count, and trading volume on major Bitcoin exchanges from *blockchain.com*, Wikipedia Bitcoin pageviews from Wikipedia *Pageviews Analysis*, and Google search trends for Bitcoin from *trends.google.com*. The Economic Policy Uncertainty Index data based on daily news are obtained from *policyuncertainty.com*. See Table A.3 in the online appendix for the complete list of predictors we consider.

4.2 Time-Dependency in Returns and Granger Causality Tests

To lay the groundwork, we first investigate how lagged Bitcoin and stock market returns predict Bitcoin returns. Each column of Table 5 Panel A shows OLS (ordinary least squares) estimates

[27] The results are practically invariant even if we use $\Delta\rho_t$ or an alternative predictor $\Delta w_{b,(t-1):t}^{(cor|vol)} = w_{b,t}(\overline{\mu}^* = 1, \sigma_{t-1}^*, \rho_t) - w_{b,t}(\overline{\mu}^* = 1, \sigma_{t-1}^*, \rho_{t-1})$

[28] The US stock market closes at 4pm Eastern Standard Time (EST) whereas the Bitcoin market closes at 12am Coordinated Universal Time (UTC). Bitcoin closing price is recorded 3 or 4 hours later than the S&P500 index on a given day depending on daylight saving. For simplicity, we ignore this time difference and use closing price of both assets in our analyses.

on $b_1^{(L)}$ and their Newey-West t-statistics of the following univariate predictive regression for the post-futures sample:

$$r_{b,t+1} = b_0 + b_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

where $r_{b,t+1}$ is daily Bitcoin log returns and z_{t+1-L} is a lagged independent variable: changes in diversification demand $\Delta w_{b,(t-1):t}^{(cor)}$, Bitcoin returns $r_{b,t}$, or S&P500 returns $r_{m,t}$ for $L = 1, ..., 4$. We observe that the coefficient is large (0.51) and significant (t-statistics = 4.15) only for $\Delta w_{b,(t-1):t}^{(cor)}$, that is, the lagged Bitcoin demand change due to correlations with lag $L = 1$. The coefficient reduces to less than its half (0.21) at the second lag $L = 2$ but its size is still the second largest among all in Panel A. The decreasing pattern in coefficients is consistent with the sequential learning effect that gradually fades.

We conduct the Granger causality tests (Granger 1969) in multivariate time-series setting:

$$r_{b,t+1} = b_0 + \sum_{L=1}^{4} b_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

Table 5 Panel B reports three different regression results. First, we include lagged changes in correlation $\Delta w_{b,(t-1):t}^{(cor)}$ and lagged Bitcoin returns. Second, we add lagged S&P500 returns to the first case. Finally, we repeat the second case using the weighted least squares (WLS) with weights that equal the inverses of time-varying variance estimates from DCC(1,1)-GARCH(1,1). We confirm that the same pattern as the univariate regressions in Panel A appears in all cases of the Granger causality tests in Panel B. Lagged Bitcoin returns and lagged S&P500 returns fail to predict Bitcoin returns in any case.

4.3 In-Sample Analysis with Other Predictors

We consider the following predictive regression model with other variables:

$$r_{b,t+1} = b_0 + b_1 \Delta w_{b,(t-1):t}^{(cor)} + Z_t \gamma + \varepsilon_{t+1}, \tag{10}$$

where b_1 measures how subsequent Bitcoin price reacts to the lagged changes in correlation. Table 6 shows that a one-standard-deviation increase (decrease) in diversification demand changes $\Delta w_{b,(t-1):t}^{(cor)}$ predicts 0.51% higher (lower) Bitcoin returns the next day. R^2 in a univariate regression is 1.15% at daily frequency, which is larger than $R^2 = 1.05\%$ by $\Delta\rho_t$. The coefficient b_1 estimates remains stable around 0.5 and statistically significant with various control variables or without the sample after the COVID-19 outbreak.[29] Note our main predictor $\Delta w^{(cor)}_{b,(t-1):t}$ is a time difference and almost serially-uncorrelated; the first-order autocorrelation is -0.06 (-0.01 before COVID-19). Thus, our predictive regression is free from the common bias in predictive regressions (Stambaugh 1999).

By contrast, time-varying daily CAPM β_t, constructed by DCC-GARCH estimates, barely affects next-day Bitcoin returns. The coefficient on lagged Bitcoin returns is small, -0.42, and insignificant (t-statistics between -1.72 and -1.77). Lagged trading volume of Bitcoin negatively predicts Bitcoin returns but becomes statistically insignificant when other Bitcoin attributes are included as controls in regression. Economic Policy Uncertainty (EPU) Index with a coefficient estimate around 0.63 is positively associated with subsequent Bitcoin returns but shy of statistical significance (t-statistics between 1.67 and 1.69). Table A.3 in the online appendix provides the complete list of control variables included in Table 6.

4.4 Out-of-Sample Predictability

We perform out-of-sample tests to address potential issues in the in-sample estimation: over-fitting, look-ahead biases, and coefficient instability. We re-estimate the DCC(1,1)-GARCH(1,1) and fit predictive regression models every trading day in real time to forecast sequentially Bitcoin returns using $\Delta\rho_t$ without any look-ahead bias. We increase the estimation (training) window after initial training with one year of data such that our prediction period starts from the beginning of 2019.

Table 7 shows the out-of-sample performances of OLS (Ordinary Least Squares), WLS (Weighted Least Squares), LAD (Least Absolute Deviation), and Rank regressions. We report two performance metrics in addition to the corresponding in-sample R^2. Campbell and Thompson (2008) suggest out-of-sample R^2 statistics:

$$R^2_{OS} = 1 - \frac{SSE(p)}{SSE(h)}, \tag{11}$$

where $SSE(p)$ is the sum of squared forecast errors by our real-time predictive regression model

[29] We do not report here, but the results are practically invariant with other linear models such as WLS (Weighted Least Squares), LAD (Least Absolute Deviation), and Rank regressions.

whereas $SSE(h)$ is by the historical average of the past Bitcoin returns as a benchmark forecast. Positive R^2_{OS} is regarded as evidence for return-predictability beyond that by the benchmark forecast.

While R^2_{OS} uses historical average returns as a benchmark, Gu, Kelly, and Xiu (2020) argue that this benchmark is flawed when analyzing individual stock returns with noisy historical averages. To deal with that, they propose a modified out-of-sample performance measure, denoted as R^{2*}_{OS}, that compares prediction against a forecast value of zero:

$$R^{2*}_{OS} = 1 - \frac{SSE(p)}{SSE(0)} \tag{12}$$

In other words, $SSE(0)$ in R^{2*}_{OS} uses flat zero as a benchmark forecast rather than historical average returns. In our test sample, R^{2*}_{OS} is lower than R^2_{OS}, which means the historical average performs worse than *zero as a forecast* for Bitcoin returns.

In general, R^2_{OS} is typically lower than in-sample R^2. We find the same pattern in the ordinary least-squares estimation (OLS) in Table 7. However, our robust alternatives (WLS, LAD and Rank regressions) even outperform the in-sample predictions. Even though R^{2*}_{OS} is generally lower than R^2_{OS}, it largely outperforms in-sample R^2 for median and rank-based estimation (LAD and Rank). Therefore, concerns about potential over-fitting and coefficient instability are well cleared. Also, outperforming out-of-sample forecasts suggest that investors may sequentially learn about the parameters in the time-series model in addition to the time-varying correlations. Strong out-of-sample predictability by robust linear regression models also suggests that our return predictability is not driven by outliers or influential points since such methods are less sensitive to the extreme observations. Also, excluding the COVID-19 period sample barely changes the predictability.

This out-of-sample predictability evidence is impressive even to practitioners. Campbell and Thompson (2008) argue that even a small R^2_{OS}, such as 0.5% at monthly frequency, can deliver economically meaningful return predictability to investors, let alone at daily frequency. Therefore, this out-of-sample Bitcoin return predictability offers a valuable trading opportunity to investors in practice.

5 Discussion

This section provides additional discussions and results to resolve remaining concerns from the previous sections.

5.1 Calibrate Free Parameters in the Model

The model in Section 3 has two free parameters that are difficult to calibrate using the data: $w_{b,t-1}^{(a)}$ and $Q_{b,t-1}$. However, we can make a statistical inference on them through the model as a lens. That is, assuming the model in Section 3 is correct, we can translate the confidence interval of a_1 coefficient in Table 2 into that of $(w_{b,t-1}^{(a)}, Q_{b,t-1})$. Figure A.1 in the online appendix shows 95% confidence region of $(w_{b,t-1}^{(a)}, Q_{b,t-1})$. As expected, the interval includes $(w_{b,t-1}^{(a)} = 0.6, Q_{b,t-1}^{(a)} = 0.3)$ that produces $a_1 = -0.154$. We admit that the information from a single parameter a_1 cannot identify two parameters $(w_{b,t-1}^{(a)}, Q_{b,t-1})$; therefore, these two free parameters are not fully identified in our parsimonious model.

5.2 Placebo Test

If passive Bitcoin investors constantly monitor the correlation dynamics, they should notice when correlation is time-varying and when it is not. If the true correlation is constant for certain period, passive investors should not respond to correlation-change-like observations. For example, even if the true correlation is zero and constant, returns of the two assets can have the same sign several times in a row, but this event should not be recognized as an increase in correlation. In a DCC-GARCH frame work (Equation 1), correlation is constant if $a = 0$. But if a passive investor falsely uses $a > 0$, then she will recognize this event as an increase in correlation.

Table 8 suggests how we can exploit a structural break in correlation dynamics for a placebo test.[30] We separately fit DCC-GARCH models to three periods: 1) from 01/01/2015 to 12/31/2020 (full sample, henceforth), 2) from 01/01/2015 to 12/17/2017 (pre-futures sample, henceforth), and

[30] Co-occurrence of the structural break and futures trading is interesting and deserves deeper investigations in both theoretical and empirical aspects. However, the results and conclusion of our analysis do not depend on the causal link between the structural break and futures trading. Therefore, finding the root cause of the structural break is beyond the scope of this paper, let alone various forces that exist behind the observed structural break. For example, changes in overall money supply or fund flows can generate positive correlations across different asset classes. Co-varying risk appetites or speculative sentiments can do the same. Yet, a rising demand of safe haven assets can generate negative correlations through a self-fulfilling prophecy. If each of these factors acts dominantly in different times, the correlation between Bitcoin and the stock market can be time-varying.

3) from 12/18/2017 to 12/31/2020 (post-futures sample, henceforth). In the table, the parameter a in DCC(1,1)-GARCH(1,1) model is zero in the pre-futures sample but 0.030 (t-stat 2.01) in the post-futures sample. The estimated conditional correlations are practically zero and flat for the pre-futures sample while the correlations fluctuate for the post-futures sample.

If our hypothesis in Section 3.1 is correct, we should observe absence of predictability when the correlation is practically constant. Instead, the return predictability should appear even for a constant-correlation period if an alternative mechanism explains the observed return predictability or if investors fail to recognize true correlation dynamics.

To test this idea, we generate illusory time-varying correlations for the pre-futures sample, by using the DCC-GARCH estimates from the full-sample, as if investors incorrectly use the full-sample parameters to track the time-varying correlation. As expected, return predictability disappear in the pre-futures period when the true correlation is constant. Table 9 shows that the coefficient on $\Delta\rho_t$ for the pre-futures sample is small and insignificant, ranging from -0.07 to -0.10 with t-statistics around -0.6. Therefore, this placebo test supports our hypothesis in Section 3.1.

5.3 Robustness Tests

We perform extensive robustness tests to solidify the return predictability evidence. First, we use other types of multivariate GARCH models to estimate time-varying correlations. Second, we run quantile regressions at different quantiles to confirm robustness against non-normality and extreme observations. Third, we trim or winsorize the data since Bitcoin returns are highly volatile and fat-tailed. Lastly, we use machine learning techniques to check how interactions between competing predictors and nonlinearity can affect the results. We use LASSO, Group Lasso, Elastic Net, Random Forest, Gradient Boosting Machine, and Neural Network with one hidden layer. We find that the return-predictability evidence is highly robust and practically invariant to all these alternative specifications. We provide the detailed results in the online appendix A.3.

6 Conclusion

Bitcoin offers a unique opportunity to understand the asset pricing implications of heterogeneous investors with different beliefs (optimism vs. pessimism) and different investment objectives (price appreciation vs. diversification). The degree of such heterogeneity in Bitcoin markets is so large that strong daily return predictability is observed, in contrast to traditional safe-haven asset markets. Furthermore, Bitcoin opens a gate for new class of assets, e.g., NFT (non-fungible token). We expect that new assets with different characteristics will also provide an opportunity to reveal new asset pricing implications as Bitcoin does in this paper. Therefore, this study sets an example for the future researches with new assets on the way.

Note that the return predictability in this paper differs from that of stock markets in the literature. Above all, Bitcoin's daily return predictability in the paper is unrelated to time-varying risk premium. Instead, passive investors' incentive to monitor the correlation plays a crucial role in predictability. Therefore, this predictability is likely to fade away if the correlation becomes no longer time-varying or if Bitcoin can no longer attract passive investors with diversification demands. However, the main idea of the paper is not only about correlation *per se* or diversification benefits. When a new asset is uncertain in various aspects, investors with different perspectives can seek what they want from the same asset even if they want different benefits from it. If Bitcoin remains highly uncertain in other aspects and still offers various benefits, we can expect another predictability or research opportunity from Bitcoin.

However, we admit that our analysis has some limitations. First, the stylized model in the paper is purely static; it does not internalize investors' learning dynamics in equilibrium. However, building a full dynamic model is a challenge because the model should also internalize all other non-conventional aspects of Bitcoin pricing, such as gradual adoption, mining cost, a fork, halving, and sentiments. Such a unified model would be too complicated to analyze and test. Second, our model is based on the assumption of mean-variance investors. However, Bitcoin is well known for its high skewness and kurtosis. The first two moments have certainly a major effect, yet considering higher moments might lead to other interesting findings. Finally, we do not explain why correlations start to fluctuate as soon as Bitcoin futures trading begins. Also, we do not build a model that can endogenously generate time-varying correlation. We only use such a structural break to split the sample for a main analysis (with post-futures sample) and a placebo test (with pre-futures sample). We leave these unanswered topics for future research.

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Figure 1: Time-Varying Correlations of Safe-Haven Assets with S&P500 Index

The figure shows the time-varying correlation estimates from DCC(1,1)-GARCH(1,1) models with daily log returns of S&P500 Index and a safe-haven asset: Bitcoin (solid black), gold (dashed black), long-term treasuries (solid grey with circles).

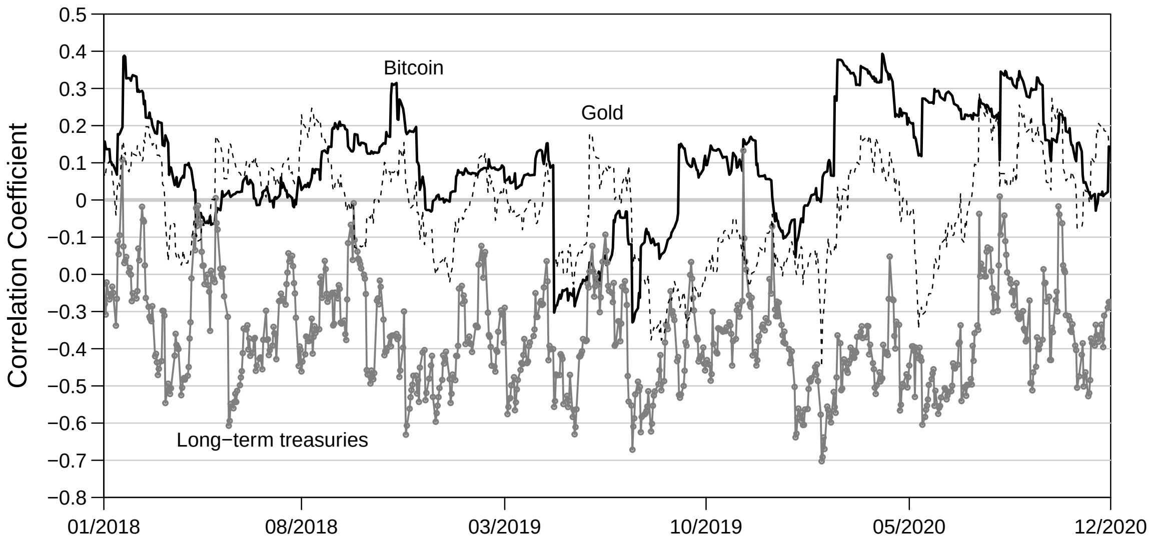


Figure 2: Comparative Statics Analysis of the Bitcoin Pricing Model

Panel (a) shows how Bitcoin price changes when the non-speculative investors' estimate on the correlation changes at static equilibrium as a comparative static analysis. The vertical axis is the subsequent log returns (%) of Bitcoin, in excess of log returns without correlation changes. Panel (b) repeats Panel (a) but replaces the horizontal axis by changes in non-speculative investors' optimal portfolio weights (%) on Bitcoin, corresponding to the correlation changes in Panel (a). We calibrate model parameters following Table 3, including $w_{b,t-1}^{(a)} = 0.6$ and $Q_{b,t-1}^{(a)}/Q_{b,t-1} = 0.3$.

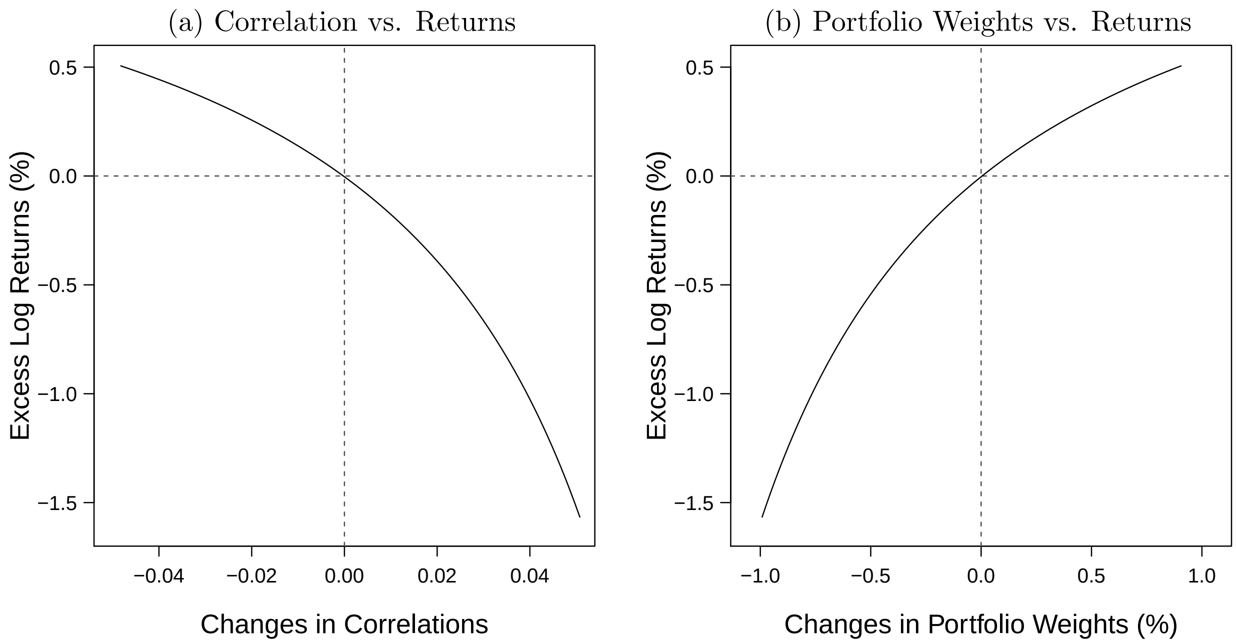


Figure 3: Model Implied Coefficient in Predictive Regressions (I)

This figure visualizes Table 3 using a finer grid of $w^{(a)}_{b,t-1}$ and $Q^{(a)}_{b,t-1}$. The observed coefficient $a_1 = -0.153$ is about 25.4[th] percentile of the square area in the plot. That is, the lower left triangular area comprises 25.4% of the whole square in the plot, and the model-implied coefficient is smaller than -0.153 in that area.

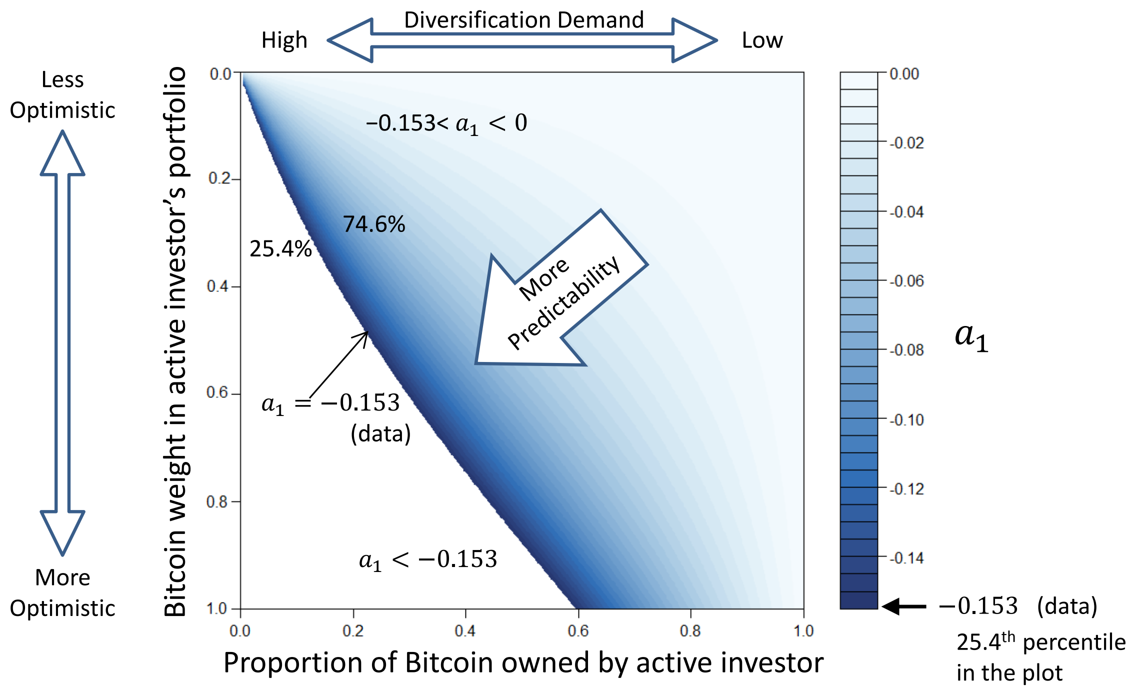


Figure 4: Model Implied Coefficient in Predictive Regressions (II)

The figure shows the model-implied coefficient a_1 in Equation (9) for different levels of correlation $(+0.1, 0, -0.05)$ and volatility ratio (σ_b/σ_m) ranging from zero to 5.5. The 25.4$^{\text{th}}$ percentile of a_1 is chosen so that the model is tuned to match the Bitcoin's return predictability result. Then it is calculated from a plot like Figure 3 for each combination of correlation and volatility ratio (σ_b/σ_m).

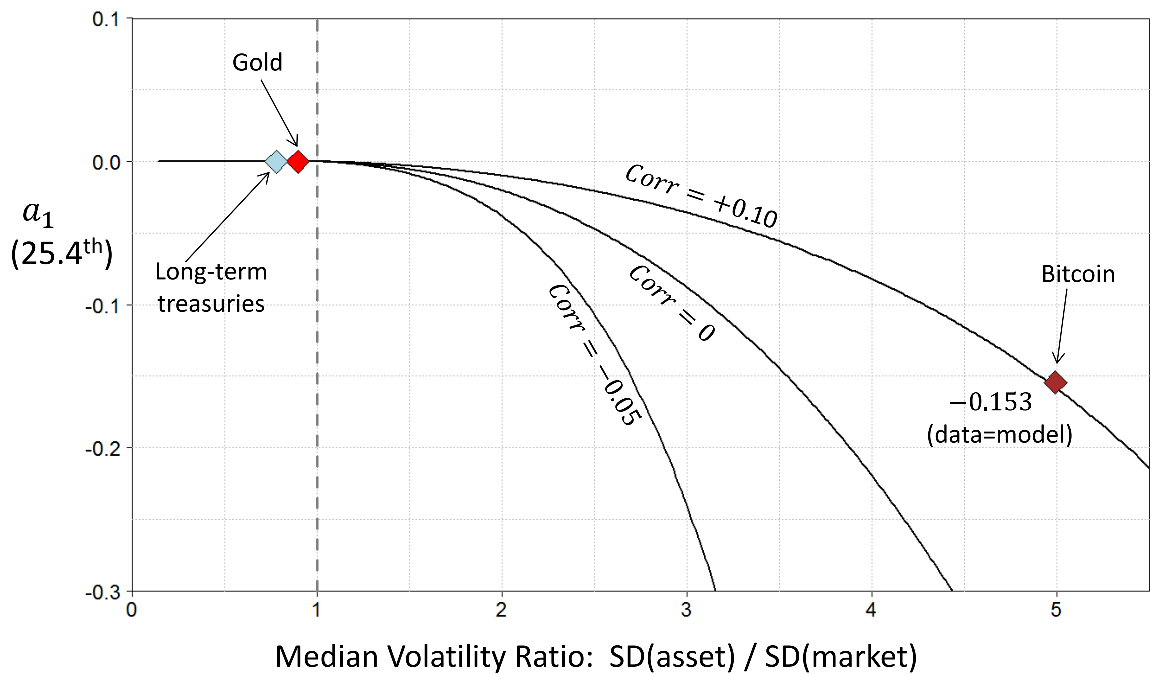


Table 1: Selected Parameter Estimates of DCC-GARCH Models

This table shows the estimates of parameters a and b and associated t-statistics (in parenthesis) by DCC(1,1)-GARCH(1,1) model for Bitcoin, gold, and long-term treasuries when they are paired with S&P500 index, respectively. The sample period ranges from 12/18/2017 to 12/31/2020.

	Bitcoin	Gold	Long-term treasuries
DCC(1,1)			
a	0.030	0.043	0.083
	(2.01)	(2.19)	(3.39)
b	0.948	0.916	0.796
	(30.84)	(28.49)	(13.36)

Table 2: Return Predictability of Safe-Haven Assets

This table shows the results of univariate predictive regressions in Equation (2):

$$r_{b,t+1} = a_0 + a_1 x_t + \varepsilon_{t+1}$$

where $r_{b,t+1}$ is daily Bitcoin log returns and x_t can be any variable in the following. ρ_t is lagged Bitcoin-S&P500 correlation level estimated from DCC(1,1)-GARCH(1,1), β_t is lagged CAPM β, cov_t is lagged Bitcoin-S&P500 covariance, and $\Delta\rho_t$ is a lagged change in correlation. We compute β_t and cov_t using correlation and volatility estimates from DCC(1,1)-GARCH(1,1). All predictor variables x_t are not standardized. 'pre-COVID' indicates that regression is performed with data from 12/18/2017 to 02/29/2020. 'LAD' is short for least absolute deviations regression and 'Rank' refers to a rank-based estimation (Hettmansperger and McKean 2010). 'INT' means both dependent variable and predictor adopt Inverse Normal Transformation (INT) based on Beasley, Erickson, and Allison (2009). 'Trimmed' indicates that both dependent variable and predictor are trimmed at 2.5% and 97.5%. All t-statistics are Newey West t-statistics except for LAD and Rank where original t-statistics are used.

Predictor x_t	coefficient a_1	t-statistic (NW)	R^2 (%)
1. Gold-S&P500			
$\Delta\rho_t$	-0.005	-0.487	0.054
2. VGLT-S&P500			
$\Delta\rho_t$ (VGLT-S&P500)	-0.003	-0.534	0.049
3. Bitcoin-S&P500			
$\Delta\rho_t$	-0.153	-3.682	1.055
$\Delta\rho_t$ (pre-COVID)	-0.153	-2.888	1.079
$\Delta\rho_t$ (LAD)	-0.129	-3.816	1.017
$\Delta\rho_t$ (Rank)	-0.142	-4.085	1.039
$\Delta\rho_t$ (INT)	-0.134	-3.598	1.787
$\Delta\rho_t$ (Trimmed)	-0.245	-2.824	1.255
ρ_t	0.001	0.049	0.000
β_t	-0.002	-0.735	0.063
cov_t	2.682	0.664	0.063

Table 3: Model-Implied Coefficients in Predictive Regressions (Corr. Changes)

This table shows the model-implied coefficient:

$$a_1 = \frac{1}{P_{b,t}} E\left[\frac{\partial P_{b,t+1}}{\partial \rho_t}\right] = E\left[\frac{\partial \log P_{b,t+1}}{\partial \rho_t}\right],$$

which corresponds to the slope coefficient a_1 in the predictive regression of Bitcoin returns $r_{b,t+1}$ on $(\rho_t - \rho_{t-1})$, i.e., changes in correlation between Bitcoin and the stock market returns:

$$r_{b,t+1} = a_0 + a_1(\rho_t - \rho_{t-1}) + \varepsilon_{t+1}$$

We compute a_1 using a numerical derivative for each combination of $w^{(a)}_{b,t-1}$ and $Q^{(p)}_{b,t-1}/Q_{b,t-1}$. At day $t-1$, speculative investors have the optimal weight on Bitcoin $w^{(a)}_{b,t-1}$ at $t-1$. Also, at $t-1$, speculative and non-speculative investors who participate in trading at t hold $Q^{(a)}_{b,t-1}$ and $Q^{(p)}_{b,t-1}$ units of Bitcoin, respectively, where $Q_{b,t-1} = Q^{(a)}_{b,t-1} + Q^{(p)}_{b,t-1}$. The point estimate on a_1 of the regression using our data is -0.153 where its 95% confidence interval implied by Newey-West standard errors is $(-0.236, -0.072)$. Therefore, we can reverse-engineer two parameters $w^{(a)}_{b,t-1}$ and $Q^{(a)}_{b,t-1}/Q_{b,t-1}$ by matching the point estimate to the numbers in the table. For example, calibrating $w^{(a)}_{b,t-1} = 0.6$ and $Q^{(p)}_{b,t-1}/Q_{b,t-1} = 0.3$ produces $a_1 = -0.154$ if other parameters are calibrated at their empirically representative values as in Section 3.3.

	$Q^{(a)}_{b,t-1}/Q_{b,t-1}$								
$w^{(a)}_{b,t-1}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	-0.053	-0.024	-0.014	-0.009	-0.006	-0.004	-0.003	-0.001	-0.001
0.2	-0.117	-0.053	-0.031	-0.020	-0.013	-0.009	-0.006	-0.003	-0.001
0.3	-0.195	-0.088	-0.052	-0.033	-0.022	-0.015	-0.010	-0.006	-0.002
0.4	-0.291	-0.133	-0.078	-0.051	-0.034	-0.023	-0.015	-0.009	-0.004
0.5	-0.408	-0.189	-0.112	-0.072	-0.048	-0.032	-0.021	-0.012	-0.005
0.6	-0.554	-0.260	-0.154	-0.100	-0.067	-0.045	-0.029	-0.017	-0.008
0.7	-0.736	-0.350	-0.209	-0.136	-0.091	-0.061	-0.040	-0.023	-0.010
0.8	-0.964	-0.467	-0.281	-0.184	-0.124	-0.083	-0.054	-0.031	-0.014
0.9	-1.253	-0.621	-0.377	-0.248	-0.167	-0.113	-0.073	-0.043	-0.019

Table 4: Coefficient in Predictive Regressions

This table compares the model-implied coefficient a_1 in Equation (9) with the estimated coefficient a_1 in Equation (2). In case of Bitcoin, the estimate $a_1 = -0.153$ (from Table 2) matches the model-implied coefficient at its 25.4$^{\text{th}}$ percentile. Two unconditional estimates $(\overline{\rho}, \sigma^*)$ are the medians of (ρ_t, σ_t^*), from the fitted DCC(1,1)-GARCH(1,1) reported in Table 1, respectively. "Data" column repeats estimates for a_1 (in Table 2).

Safe-Haven Asset	$\overline{\rho}$	σ^*	Coefficient a_1				
			Model-implied			Data	t-stat.
			37.5$^{\text{th}}$	25.4$^{\text{th}}$	12.5$^{\text{th}}$		
Bitcoin	0.095	4.993	-0.078	-0.153	-0.396	-0.153	(-3.682)
Gold	0.000	0.895	-0.000	-0.000	-0.000	-0.005	(-0.487)
Long-Term Treasuries	-0.376	0.779	-0.000	-0.000	-0.000	-0.003	(-0.534)

Table 5: Time-Dependency in Returns and Granger Causality Tests

Panel A shows OLS (ordinary least squares) estimates on the slope coefficient $a_1^{(L)}$ and Newey-West t-statistics of the following univariate predictive regression for the post-futures sample:

$$r_{b,t+1} = b_0 + b_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

where $r_{b,t+1}$ is daily Bitcoin log returns (%) and z_{t+1-L} is a lagged independent variable: Bitcoin demand changes due to correlation $\Delta w_{b,(t-L):t}^{(cor)}$ (standardized), Bitcoin returns $r_{b,t}$, or S&P500 returns $r_{m,t}$ for $L = 1, ..., 4$. Each column in Panel A is from an individual univariate regression. On the other hand, Panel B shows three Granger causality test results in multivariate time-series setting:

$$r_{b,t+1} = b_0 + \sum_{L=1}^{4} b_1^{(L)} z_{t+1-L} + \varepsilon_{t+1},$$

WLS refers to weighted least squares with weights that equal the inverses of time-varying variance estimates from DCC(1,1)-GARCH(1,1) fitting.

	$\Delta w_{(t-L):(t-L+1)}^{(cor)}$				Bitcoin returns				S&P500 returns			
L (lag)	1	2	3	4	1	2	3	4	1	2	3	4
Panel A: Univariate time-series regression												
$a_1^{(L)}$	0.51	0.21	-0.13	0.01	-0.07	0.08	0.05	-0.01	-0.15	0.10	0.02	-0.02
t-stat	4.15	0.48	-0.66	0.06	-1.49	1.87	1.31	-0.20	-1.00	1.07	0.11	-0.20
Panel B: Granger causality tests												
OLS coef.	0.43	0.21	-0.14	-0.07	-0.08	0.06	0.05	0.00				
t-stat	2.68	0.47	-0.82	-0.36	-1.76	1.16	1.25	0.07				
OLS coef.	0.42	0.21	-0.13	-0.08	-0.07	0.05	0.05	0.00	-0.09	0.03	0.00	-0.01
t-stat	2.64	0.49	-0.75	-0.42	-1.59	1.07	1.39	0.02	-0.67	0.30	0.01	-0.08
WLS coef.	0.65	0.25	-0.07	-0.06	0.01	0.04	0.07	0.02	0.13	0.06	-0.02	-0.07
t-stat	3.23	1.26	-0.33	-0.32	0.21	0.76	1.53	0.55	0.99	0.42	-0.19	-0.57

Table 6: In-Sample Predictive Regressions for Bitcoin

The table shows the coefficient estimates and associated robust t-statistics in the predictive regression (10):

$$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + Z_t \gamma + \varepsilon_{t+1},$$

where $r_{b,t+1}$ is daily Bitcoin log returns (%). $\Delta w^{(cor)}_{b,(t-1):t}$ refers to (standardized) changes in non-speculative Bitcoin demand due to correlation changes from $t-1$ to t, respectively. Robust t-statistics in parenthesis are calculated from Newey and West (1987) and Newey and West (1994). Other predictor variables Z_t include lagged Bitcoin returns $r_{b,t} = \log(P_{b,t}) - \log(P_{b,t-1})$, lagged CAPM beta β_t, lagged Bitcoin trade volume $Volume_{b,t}$, Economic Policy Uncertainty index (policyuncertainty.com) $EPU_{b,t}$, and additional control variable groups. In Table A.3 in the oniline appendix, column 'CG', we classify each additional control variable into one of the following groups: Group M (market attributes), Group B (blockchain attributes), and Group L (extra lagged returns). When including a control variable group in regression, we use the first two principal components from that group. Note that investor attention variables related to Google search trends for Bitcoin are based on Liu and Tsyvinski (2021). Group L includes $r_{b,t-L}$ for $L = 1, ..., 4$ and $r_{m,t-L}$ for $L = 0, ..., 4$, implying Granger causality test.

Predictor	Post-futures (12/18/2017 to 12/31/2020)				Post-futures before COVID-19 (12/18/2017 to 02/29/2020)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta w^{(cor)}_{b,(t-1):t}$	0.51	0.47	0.47	0.47	0.50	0.47	0.47	0.48
	(4.15)	(2.89)	(2.83)	(2.80)	(3.16)	(2.76)	(2.61)	(2.37)
$r_{b,t}$		-0.42	-0.42	-0.42		-0.22	-0.22	-0.22
		(-1.77)	(-1.73)	(-1.72)		(-0.81)	(-0.80)	(-0.80)
β_t		0.04	0.06	0.06		0.06	0.07	0.09
		(0.21)	(0.29)	(0.29)		(0.27)	(0.31)	(0.38)
$Volume_{b,t}$		-0.58	-0.55	-0.55		-0.73	-0.91	-0.86
		(-2.59)	(-1.37)	(-1.43)		(-3.48)	(-1.93)	(-1.76)
$EPU_{b,t}$		0.62	0.63	0.62		0.21	0.20	0.21
		(1.67)	(1.68)	(1.69)		(0.98)	(0.93)	(0.96)
Controls		M	MB	MBL		M	MB	MBL
R^2 (%)	1.15	3.94	3.98	4.00	1.17	3.80	3.91	4.20

Table 7: Out-of-Sample Predictive Regressions

This table repeats column (1) of Table 6 in a completely out-of-sample fashion. The reported numbers are in- and out-of-sample performance measures of the predictive regression:

$$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + \varepsilon_{t+1}.$$

At each day t, we re-estimate $\Delta x_{b,(t-1):t}$ (from DCC-GARCH) and (b_0, b_1) while expanding a training window without any look-ahead biases. We compare four different estimation methods: ordinary least squares (OLS), weighted least squares (WLS), least absolute deviation (LAD, median), and rank-based estimation (Rank) from Hettmansperger and McKean (2010). For WLS weights, we use reciprocals of real-time time-varying variance estimates from DCC(1,1)-GARCH(1,1). R^2_{IS} is in-sample R^2. R^2_{OS} is defined in Equation (11) following Campbell and Thompson (2008) while R^{2*}_{OS} is defined in Equation (12) following Gu, Kelly, and Xiu (2020).

Evaluation period	1/2/2019 to 12/31/2020				1/2/2019 to 2/29/2020			
	OLS	WLS	LAD	Rank	OLS	WLS	LAD	Rank
$\Delta w^{(cor)}_{b,(t-1):t}$, from S&P500								
R^2_{IS} (%)	1.29	1.17	0.92	0.97	1.47	1.36	1.07	1.07
R^2_{OS} (%)	1.07	1.54	2.32	2.33	1.49	2.07	3.49	3.52
R^{2*}_{OS} (%)	-0.08	0.40	1.19	1.20	-0.55	0.04	1.49	1.52

Table 8: Selected Parameter Estimates of DCC-GARCH Models in Sub-Periods

This table shows the estimates of parameters a and b and associated t-statistics in parenthesis by DCC(1,1)-GARCH(1,1) model with Bitcoin and S&P500 for the pre-futures, the post-futures, and the full sample period. A full set of parameter estimates are reported in the online appendix A.3.1.

Sample	Pre-futures	Post-futures	Full
from	01/01/2015	12/18/2017	01/01/2015
to	12/17/2017	12/31/2020	12/31/2020
DCC(1,1)			
a	0.000	0.030	0.016
	(0.00)	(2.01)	(1.52)
b	0.925	0.948	0.964
	(2.94)	(30.84)	(28.84)

Table 9: Predictive Regressions for Bitcoin in Sub-Periods

This table repeats columns (1) of Table 6 for three different sample periods: full (01/01/2015 to 12/31/2020), pre-futures (01/01/2015 to 12/17/2017) and post-futures (12/18/2017 to 12/31/2020). The reported numbers are coefficients and associated Newey-West t-statistics (in parenthesis). $\mathrm{I}^{(pre)}$ is an indicator function whose value takes one for the pre-futures sample and zero otherwise whereas $\mathrm{I}^{(post)}$ is similar but for the post-futures sample. For the first two columns, $\Delta w^{(cor)}_{b,(t-1):t}$ is estimated using the full sample while using only post-futures sample for the last column.

	Sample	Pre-futures	Post-futures	Full
	from	01/01/2015	12/18/2017	01/01/2015
Predictor	to	12/17/2017	12/31/2020	12/31/2020
$\Delta w^{(cor)}_{b,(t-1):t}\mathrm{I}^{(pre)}$		-0.10		-0.10
		(-0.52)		(-0.72)
$\Delta w^{(cor)}_{b,(t-1):t}\mathrm{I}^{(post)}$			0.51	0.52
			(4.15)	(4.35)
R^2 (%)		0.06	1.15	0.68

Online Appendix

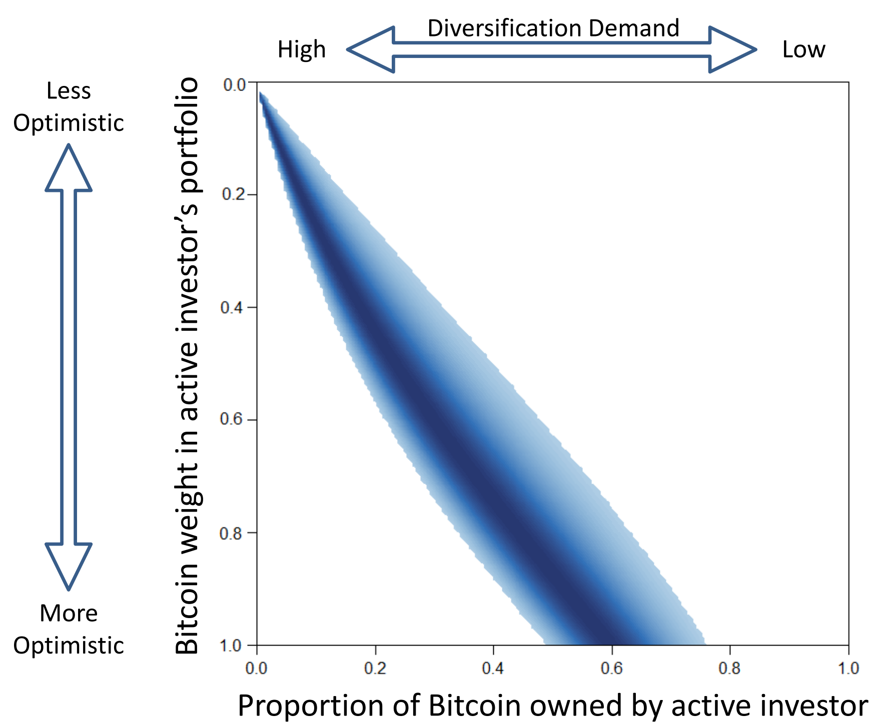
for

Volatile Safe-Haven Asset

A.1 95% Confidence Interval of $(w^{(a)}_{b,t-1},\ Q_{b,t-1})$

Figure A.1: Model Implications II

The figure shows 95% confidence interval of $(w^{(a)}_{b,t-1},\ Q_{b,t-1})$ that matches that of the coefficient estimate (a_1) in the predictive regression for Bitcoin in Table 2.



A.2 Model-Implied Return-Predictability of Correlation Level

Table A.1 shows the model-implied coefficient $a_1^{(l)}$ in predictive regressions of Bitcoin returns $r_{b,t+1}$ on ρ_t, i.e., the level of correlation between Bitcoin and the stock market returns:

$$r_{b,t+1} = a_0^{(l)} + a_1^{(l)} \rho_t + \varepsilon_{t+1} \quad \text{where} \quad a_1^{(l)} = \frac{E[r_{b,t+1}^{(+)}] - E[r_{b,t+1}^{(-)}]}{\rho_t^{(+)} - \rho_t^{(-)}},$$

where $\rho_t^{(+)} = \overline{\rho}^* + SD(\rho_t)$, $\rho_t^{(-)} = \overline{\rho}^* - SD(\rho_t)$, $(\overline{\rho}^*, \rho_t)$ are the unconditional and conditional correlations from the DCC-GARCH estimation, respectively, and $SD(\rho_t) \approx 0.15$. On the other hand, $E[r_{b,t+1}^{(+)}]$ and $E[r_{b,t+1}^{(-)}]$ are the model-implied Bitcoin log returns corresponding to the correlation level $\rho_t^{(+)}$ and $\rho_t^{(-)}$, respectively. To compute $E[r_{b,t+1}^{(+)}]$, we first find $E[\rho_{t+1}|\rho_t^{(+)}]$ from Equation (1). Then compute the equilibrium Bitcoin prices at $t+1$ when $\rho_t = E[\rho_{t+1}|\rho_t^{(+)}]$ and $\rho_t = \rho_t^{(+)}$, respectively, holding others constant. Finally, we have a gross return that equals the ratio of those two prices, and its logarithm is $E[r_{b,t+1}^{(+)}]$. We compute $E[r_{b,t+1}^{(-)}]$ in a similar fashion.

The point estimate on $a_1^{(l)}$ of the regression using our data is practically zero, ($\hat{a}_1^{(l)} = 0.0006$), where its 95% confidence interval implied by Newey-West standard errors is $(-0.0946, +0.0957)$. 80.2% of the model-implied coefficient $a_1^{(l)}$ in Table A.1 fall between zero and 0.001. Most importantly, our model can simultaneously generate both a_1 and $a_1^{(l)}$ matching their empirical estimates with a common choice of $w_{b,t-1}^{(a)}$ and $Q_{b,t-1}^{(a)} = Q_{b,t-1}^{(a)}/Q_{b,t-1}$. The model confirms that the correlation level cannot predict Bitcoin returns while its change can. Therefore, these two tables quantitatively support our explanation on the Bitcoin return predictability puzzle.

A.3 Robustness Tests

A.3.1 DCC-GARCH Models

We also consider alternative GARCH models with conditional correlations. The Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) model (Nelson 1991) can capture the leverage effects in asset returns — asymmetric responses of conditional variances to negative or positive shocks. On the other hand, an asymmetric response to joint shocks in conditional correlations can be captured by the Asymmetric Generalized Dynamic Conditional Correlation (AG-DCC)

Table A.1: Model-Implied Coefficients in Predictive Regressions (Corr. Levels)

This table repeats Table 3 but instead shows the model-implied coefficient:

$$a_1^{(l)} = \frac{E[r_{b,t+1}^{(+)}] - E[r_{b,t+1}^{(-)}]}{\rho_t^{(+)} - \rho_t^{(-)}},$$

where $\rho_t^{(+)} = \overline{\rho}^* + SD(\rho_t)$, $\rho_t^{(-)} = \overline{\rho}^* - SD(\rho_t)$, $(\overline{\rho}^*, \rho_t)$ are the unconditional and conditional correlations from the DCC-GARCH estimation, respectively, and $SD(\rho_t) \approx 0.15$. On the other hand, $E[r_{b,t+1}^{(+)}]$ and $E[r_{b,t+1}^{(-)}]$ are the model-implied Bitcoin log returns corresponding to the correlation level $\rho_t^{(+)}$ and $\rho_t^{(-)}$, respectively. The coefficient $a_1^{(l)}$ corresponds to the slope coefficient in the predictive regression of Bitcoin returns $r_{b,t+1}$ on ρ_t, i.e., levels of correlation between Bitcoin and the stock market returns:

$$r_{b,t+1} = a_0^{(l)} + a_1^{(l)} \rho_t + \varepsilon_{t+1}$$

To compute $E[r_{b,t+1}^{(+)}]$, we first find $E[\rho_{t+1}|\rho_t^{(+)}]$ from Equation (1). Then compute the equilibrium Bitcoin prices at $t+1$ when $\rho_t = E[\rho_{t+1}|\rho_t^{(+)}]$ and $\rho_t = \rho_t^{(+)}$, respectively, holding others constant. Finally, we have a gross return that equals the ratio of those two prices, and its logarithm is $E[r_{b,t+1}^{(+)}]$. We compute $E[r_{b,t+1}^{(-)}]$ in a similar fashion. The point estimate on $a_1^{(l)}$ of the regression using our data is practically zero, 0.0006, where its 95% confidence interval implied by Newey-West standard errors is $(-0.0946, +0.0957)$. Therefore, the model can simultaneously generate both a_1 and $a_1^{(l)}$ matching their empirical estimates with a common choice of $w_{b,t-1}^{(a)}$ and $Q_{b,t-1}^{(a)} = Q_{b,t-1}^{(a)}/Q_{b,t-1}$.

	$Q_{b,t-1}^{(a)}/Q_{b,t-1}$								
$w_{b,t-1}^{(a)}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0006	0.0003	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
0.3	0.0011	0.0004	0.0002	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
0.4	0.0017	0.0007	0.0004	0.0002	0.0002	0.0001	0.0001	0.0000	0.0000
0.5	0.0027	0.0010	0.0006	0.0004	0.0002	0.0002	0.0001	0.0001	0.0000
0.6	0.0040	0.0015	0.0008	0.0005	0.0003	0.0002	0.0001	0.0001	0.0000
0.7	0.0059	0.0021	0.0011	0.0007	0.0004	0.0003	0.0002	0.0001	0.0000
0.8	0.0085	0.0030	0.0016	0.0010	0.0006	0.0004	0.0003	0.0001	0.0001
0.9	0.0120	0.0042	0.0022	0.0013	0.0008	0.0006	0.0003	0.0002	0.0001

model (Cappiello, Engle, and Sheppard 2006). As a special case, an Asymmetric DCC (A-DCC) model modifies the DCC equation as follows.

$$\boldsymbol{Q}_t = (\bar{\boldsymbol{R}} - a^2\bar{\boldsymbol{R}} - b^2\bar{\boldsymbol{R}} - g^2\bar{\boldsymbol{N}}) + a^2\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1} + g^2\boldsymbol{n}_{t-1}\boldsymbol{n}'_{t-1} + b^2\boldsymbol{Q}_{t-1}$$

where a, b, and g are scalar parameters; $\bar{\boldsymbol{N}} = E[\boldsymbol{n}_t\boldsymbol{n}'_t]$ and $\boldsymbol{n}_t = I[\boldsymbol{\epsilon}_t < 0] \circ \boldsymbol{\epsilon}_t$ where I[.] is an indicator function and "∘" represents Hadamard product. The following table shows that the return-predictability results are robust to different types of DCC-GARCH models.[A.1]

[A.1] Although we do not report here, we find that adding autoregressive components in the mean function of DCC-GARCH model further strengthen our results.

Table A.2: Parameter Estimates of DCC-GARCH Models

The table shows parameter estimates and associated t-statistics in parenthesis for different DCC-GARCH models. E(1,1) refers to EGARCH(1,1) and a(1,1) refers to Asymmetric Dynamic Conditional Correlation a-DCC(1,1). Panel A, B, and C are for full, pre-futures, and post-futures sample periods, respectively. Panel A and B only report DCC parameters a and b while Panel C also includes GARCH parameter estimates and information criteria. DCC-a estimates are all zeros in the pre-futures period and become positive and overall significant in the post-futures period across all DCC-GARCH models. This result implies a constant correlation in the pre-futures period and time-varying correlations in the post-futures period.

DCC order GARCH order	(1,1) (1,1)	(1,1) E(1,1)	(1,1) (2,1)	(1,1) (1,2)	a(1,1) (1,1)
Panel A: Full sample period, 01/01/2015–12/31/2020					
DCC					
a	0.016 (1.52)	0.014 (1.61)	0.016 (1.43)	0.016 (1.43)	0.015 (1.94)
b	0.964 (28.84)	0.970 (37.78)	0.963 (26.55)	0.963 (26.24)	0.960 (27.13)
Panel B: Pre-futures period, 01/01/2015–12/17/2017					
DCC					
a	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)
b	0.925 (2.94)	0.926 (2.85)	0.924 (3.77)	0.926 (3.51)	0.942 (5.53)
Panel C: Post-futures period, 12/18/2017–12/31/2020					
DCC					
a	0.030 (2.01)	0.025 (1.88)	0.034 (1.94)	0.032 (1.99)	0.028 (1.88)
b	0.948 (30.84)	0.957 (33.67)	0.941 (26.86)	0.944 (28.54)	0.948 (30.47)
g					0.007 (0.37)
GARCH-Bitcoin					
α_1	0.192 (1.96)	-0.127 (-1.17)	0.037 (0.76)	0.223 (1.80)	0.192 (1.96)
α_2			0.266 (1.66)		
β_1	0.703 (10.86)	0.826 (18.74)	0.522 (4.59)	0.1621 (0.96)	0.703 (10.85)
β_2				0.480 (3.52)	
γ_1		0.353 (1.98)			
GARCH-S&P500					
α_1	0.285 (3.55)	-0.152 (-3.06)	0.231 (1.76)	0.285 (2.14)	0.285 (3.56)
α_2			0.093 (1.09)		
β_1	0.714 (9.88)	0.936 (53.32)	0.675 (4.85)	0.715 (1.08)	0.714 (9.88)
β_2				0.000 (0.00)	
γ_1		0.450 (4.93)			
Predictive Regression:	$r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} \varepsilon_{t+1}$				
$\Delta w^{(cor)}_{b,(t-1):t}$	0.513 (4.15)	0.189 (3.61)	0.518 (4.16)	0.512 (4.14)	0.514 (4.39)
R^2 (%)	1.15	1.05	1.18	1.15	1.16

A.3.2 Quantile Regression

We repeat our main predictive regression with quantile objective functions to confirm robustness against non-normality and extreme observations. The quantile regression satisfies the following:

$$Q(\tau) = b_0 + b_1 \Delta w_{b,(t-1):t}^{(cor)} + b_2 \Delta w_{b,(t-1):t}^{(vol)}$$

where $Q(\tau)$ is $(\tau \times 100)^{\text{th}}$ percentile of Bitcoin's daily log returns $r_{b,t+1}$. $\Delta w_{b,(t-1):t}^{(cor)}$ and $\Delta w_{b,(t-1):t}^{(vol)}$ refer to non-speculative demand changes due to correlation and volatility ratio changes from $t-1$ to t, respectively. When $\tau = 0.5$, we have least absolute deviations (LAD) regressions. Figure A.2 shows how coefficient estimates b_1 and b_2 with their 95% confidence intervals varies depending on the choice of the quantile criteria τ in Panel (a) and (b), respectively.

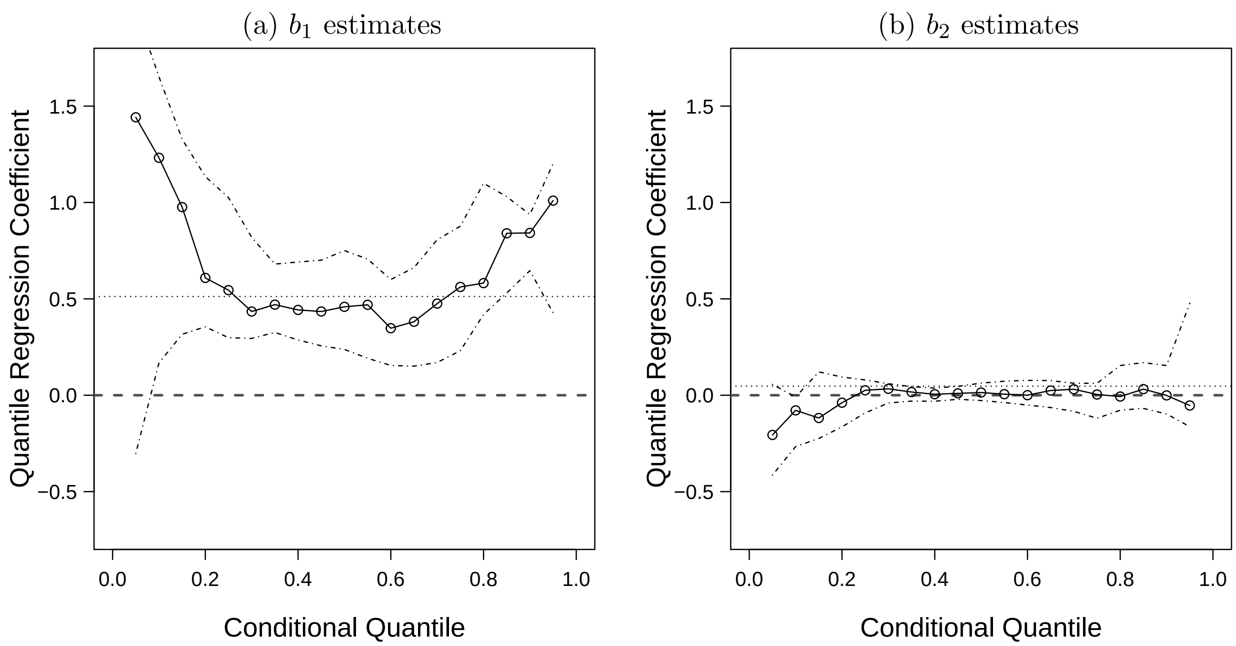
In Figure A.2, the coefficient on $\Delta w_{b,(t-1):t}^{(cor)}$, i.e., b_1, varies around the previous OLS estimate 0.5 while keeping its statistical significance for the most part, except $\tau < 0.1$. On the other hand, the coefficients on $\Delta w_{b,(t-1):t}^{(vol)}$, i.e., b_2, are rather flat around zero and insignificant over all quantiles. The quantile regression results confirm robustness of Bitcoin return predictability.

Figure A.2: Quantile Predictive Regressions

This figure shows coefficients (solid lines with circle markers) and their 95% confidence intervals (dash-dotted lines) from the quantile regressions satisfying the following:

$$Q(\tau) = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + b_2 \Delta w^{(vol)}_{b,(t-1):t}$$

where $Q(\tau)$ is $(\tau \times 100)^{\text{th}}$ percentile of Bitcoin's daily log returns $r_{b,t+1}$. $\Delta w^{(cor)}_{b,(t-1):t}$ and $\Delta w^{(vol)}_{b,(t-1):t}$ refer to non-speculative demand changes due to correlation and volatility ratio changes from $t-1$ to t, respectively. When $\tau = 0.5$, we have least absolute deviations (LAD) regressions.



A.3.3 Trimming and Winsorizing

Bitcoin returns are highly volatile and fat-tailed. A few extreme observations can distort simple OLS estimates. Here, we trim or winsorize the data to see how the extreme values affect our results in the predictive regression $r_{b,t+1} = b_0 + b_1 \Delta w^{(cor)}_{b,(t-1):t} + \varepsilon_{t+1}$. For trimming, we omit observations whenever $r_{b,t+1}$ is outside the interval $(c, 100 - c)$ percentiles. For winsorizing, we replace both dependent and independent variables outside their own intervals $(c, 100 - c)$ percentiles by their nearest boundary values. Then we fit the predictive regression and visualize the results for each cutoff point c (ranging from 0 to 5 percentiles) in Figure A.3 and A.4, for the post-futures period with and without the COVID-19 era samples. In Figure A.3, we observe that the coefficient b_1 from trimmed data is around 0.5 and statistically significant even if we omit total 10% of the sample at tails. In Figure A.4, the coefficient b_1 from the winsorized data even increases with the cutoff c.

Figure A.3: Predictive Regressions with Trimmed Data

The figure shows how different trimming schemes change OLS estimates on b_1 (solid lines with circle markers) and associated 95% confidence intervals (dash-dotted lines) from Newey-West standard errors, for two different sample periods.

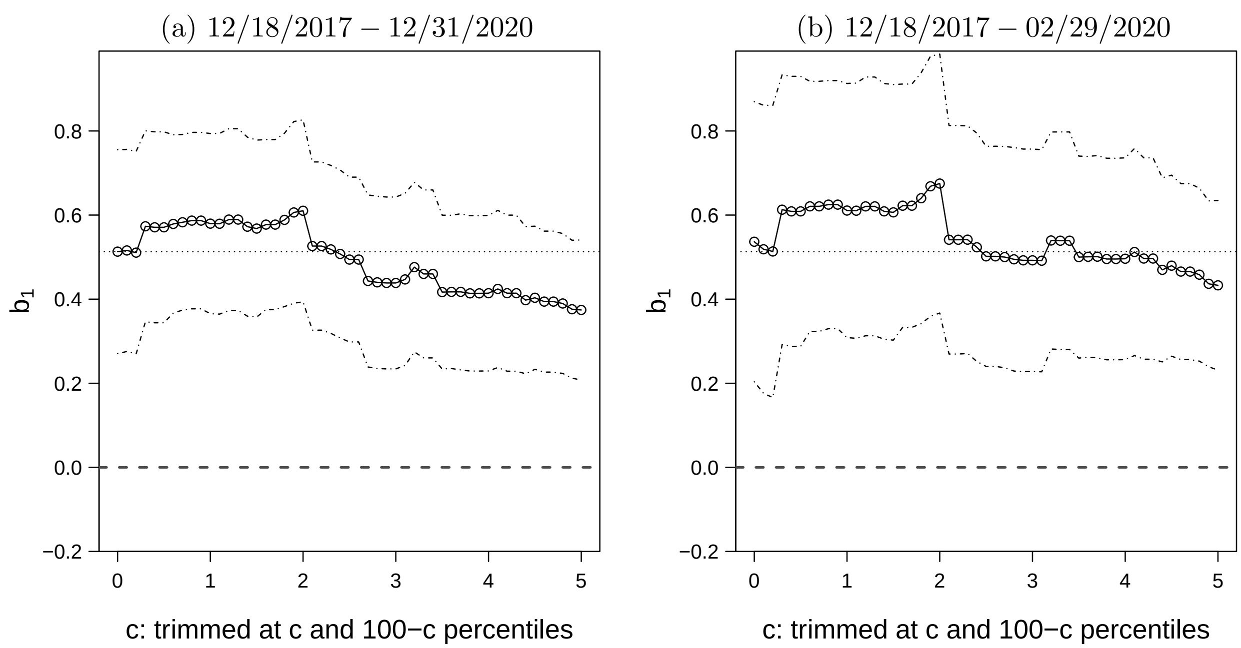
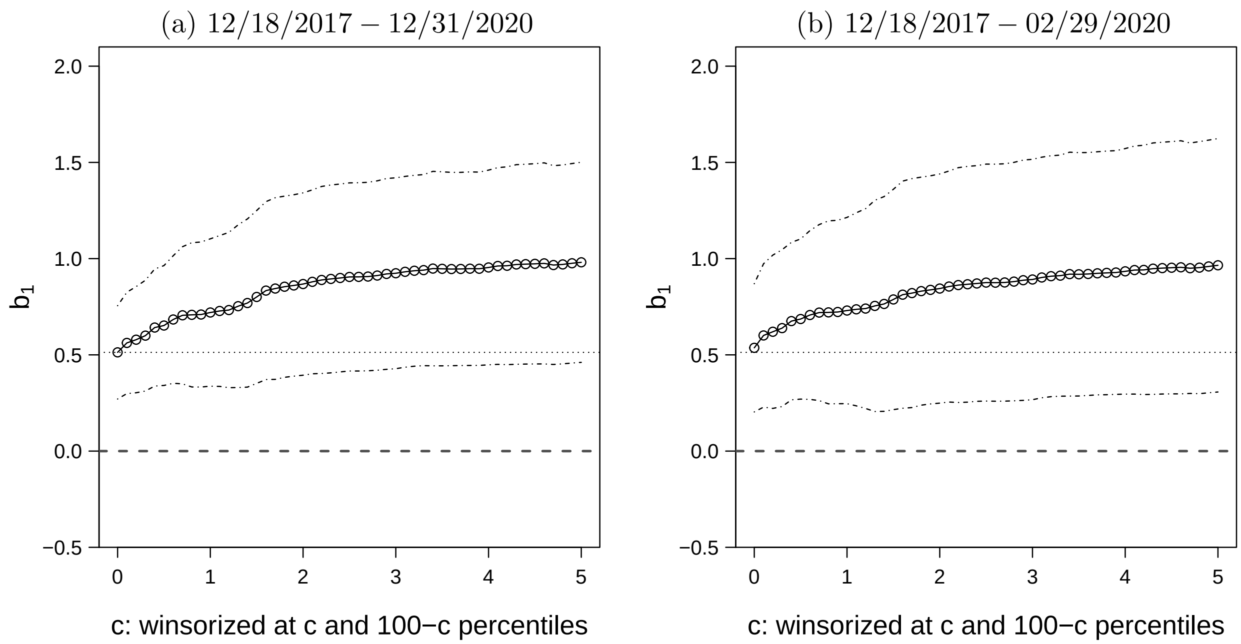


Figure A.4: Predictive Regressions with Winsorized Data

The figure shows how different winsorizing schemes change OLS estimates on b_1 (solid lines with circle markers) and associated 95% confidence intervals (dash-dotted lines) from Newey-West standard errors, for two different sample periods.



A.3.4 Relative Importance of Predictors

Changes in Bitcoin demand due to time-varying correlations certainly predict subsequent daily Bitcoin returns. However, linear regression models might suffer from over-fitting and do not clearly show how strong the predictability evidence is relative to other commonly used predictors.

To address these concerns, we apply several Machine Learning algorithms to fit the data: LASSO, Group LASSO, Elastic Net, Random Forest (RF), Gradient Boosting Machine (GBM), and Neural Network with one hidden layer (NN1). In particular, the last three algorithms consider non-linearity and all possible interactions among predictors. Table A.3 shows a list of all predictors simultaneously used in training. To avoid over-fitting, we perform a five-fold cross-validation to tune hyper-parameters for each algorithm by a grid search.[A.2]

To quantify the importance of each predictor, we compute a reduction in R^2, MSE (Mean-Squared-Errors), or MAE (Mean-Absolute-Errors) when a given predictor is set to zero in a trained model, following Gu, Kelly, and Xiu (2020). We report only the results based on R^2 since all three measures produce very similar outcomes in importance metrics. Two heat maps in Figure A.5 visualize this variable-importance metric for several machine learning algorithms. In Panel (a), the variable importance metric is scaled by the column-wise sum so that the total sum for each column can be unity. Darker colors represent greater importance in prediction. Panel (b) repeats the Panel (a) but scales the importance metric by normalizing the variable importance metric to be between zero and one for each algorithm.

Finally, Table A.4 reports the in-sample importance metric numbers for the most and the second most important predictors for each algorithm using post-futures data. The most important variable (MIV) is dominated by $\Delta w_{b,t}^{(cor)}$ and lagged trading volume; $\Delta w_{b,t}^{(cor)}$ is the top predictor in Group LASSO (GLASSO), tree-based models (RF and GBM), and an artificial neural network (NN1) while lagged trading volume is only dominant in certain linear models, LASSO and Elastic Net. It is noticeable that $\Delta w_{b,t}^{(cor)}$ is always among the top two important predictors whereas all other predictors fall short.

[A.2] We tune the shrinkage factors in LASSO, Group LASSO, and Elastic Net. For Random Forest, we tune the feature sampling size and the minimum size of terminal nodes. For Gradient Boosting Machine, we tune the number of trees, the shrinkage factor, the maximum depth of trees, the number of minimum observation in each node, and the fraction of sub-sampling. For Neural Network, we tune the L1 shrinkage factor and the learning rate. For all algorithms, we use a mean absolute error loss function for performance evaluation.

From this Machine Learning exercise, we conclude that our proposed predictor is not only orthogonal to other predictors but also a dominant predictor for daily Bitcoin returns, especially when non-linearity and possible interactions are considered.

Table A.3: A List of Predictors

The table shows a list of predictors used in Figure A.5 and Table 6 and A.4. When including both lagged change and level of the same variable, we compute the lagged level of the variable as the average of its one-day- and two-day-lagged values. Column 'CG' categorizes each variable into one of the followings: Group M (market attributes), Group B (blockchain attributes), and Group L (extra lagged returns). Column 'LG' specifies groupings of predictors for Group LASSO with those sharing an integer index belonging to a same group.

CG	LG	Predictor Name	Definition
	1	wt-cor t-1	change in optimal weight on Bitcoin from t-2 to t-1 due to time-varying correlation, $(\Delta w^{(cor)}_{b,(t-L):t})$
	3	BTC return t-1	log return of Bitcoin at time t-1
	4	CAPM Beta t-1	CAPM Beta at time t-1
	5	trade volume t-1	Bitcoin trade volume on major exchanges at time t-1
	6	EPU t-1	Economic Policy Uncertainty Index at time t
M	1	S&P500 cor t-1.5	averge of BTC-S&P500 correlation at time t-1 and t-2
M	2	S&P500 vol t-1.5	averge of S&P500 volatility at time t-1 and t-2
M	7	S&P500 return t-1	log return of S&P500 index at time t-1
M	8	diff SKEW t-1	change of COBE SKEW index from t-2 to t-1
M	8	SKEW t-1.5	average of COBE SKEW index at time t-1 and t-2
M	9	diff VIX t-1	the difference between COBE VIX index at t-1 and that at t-2
M	9	VIX t-1.5	average of COBE VIX index at time t-1 and t-2
M	10	diff S&P500 volume t-1	change of CBOE Volume for S&P 500 Index options (as % of market cap) from t-2 to t-1
M	10	S&P500 volume t-1.5	average of CBOE Volume for S&P 500 Index options at time t-1 and t-2
M	2	BTC vol t-1.5	average of Bitcoin volatility at time t-1 and t-2
M	3	BTC logprc t-1.5	average of Bitcoin log price at time t-1 and t-2
M	11	3-mon treasury yield t-1	3-month treasury yield at time t-1
M	11	10-yr treasury yield t-1	10-year treasury yield at time t-1
M	11	30-yr treasury yield t-1	30-year treasury yield at time t-1
M	12	Gold return t-1	log return of Gold at time t-1
M	13	USD return t-1	log return of US Dollar at time t-1
B	14	WK BTC t-1	Wikipedia Bitcoin search counts at time t-1
B	14	GOOG BTC t-1	Google search counts for *Bitcoin* at time t-1
B	14	GOOG BTC wavg	average Google search counts for *Bitcoin* over the past week
B	14	GOOG BTC NegAttn t-1	ratio of Google search counts for *Bitcoin hack* to *Bitcoin* at t-1
B	14	GOOG BTC NegAttn wavg	average ratio of Google search counts for *Bitcoin hack* to *Bitcoin* over the past week
B	15	transac volume t-1	Bitcoin transaction volumn at time t-1
B	16	difficulty t-1	Bitcoin mining diffficulty at time t-1
B	16	hashrate t-1	Bitcoin hashrate at time t-1
B	17	unique address t-1	number of Bitcoin unique address at time t-1
B	18	total BTC t-1	total number of Bitcoin at time t-1
B	15	unique BTC transac t-1	number of unique Bitcion transaction at time t-1
L	7	S&P500 return t-2	Daily log return of S&P500 index from t-2 to t-1
L	7	S&P500 return t-3	Daily log return of S&P500 index from t-3 to t-2
L	7	S&P500 return t-4	Daily log return of S&P500 index from t-4 to t-3
L	7	S&P500 return t-5	Daily log return of S&P500 index from t-5 to t-4
L	3	BTC return t-2	Daily log return of Bitcoin from t-2 to t-1
L	3	BTC return t-3	Daily log return of Bitcoin from t-3 to t-2
L	3	BTC return t-4	Daily log return of Bitcoin from t-4 to t-3
L	3	BTC return t-5	Daily log return of Bitcoin from t-5 to t-4

Figure A.5: Variable Importance Heat Map

Two heat maps show variable importance metrics, which are reductions in R^2 when a given predictor is set to zero in a trained model, following Gu, Kelly, and Xiu (2020). The graphs show the relative importance of each predictor in each of the following machine learning algorithms: LASSO, Group LASSO, Elastic Net, Random Forest, Gradient Boosting Machine, and Neural Network with one hidden layer. In Panel (a), the variable importance metric (Metric I) is scaled by the column-wise sum so that the total sum for each column can be unity. Darker colors represent greater importance in prediction. Panel (b) repeats Panel (a) but scales the importance metric (Metric II) by normalizing the variable importance metric to be between zero and one for each algorithm. For Group LASSO, we use the groupings specified in Table A.3 column 'LG' such that predictors with similar characteristics are grouped together.

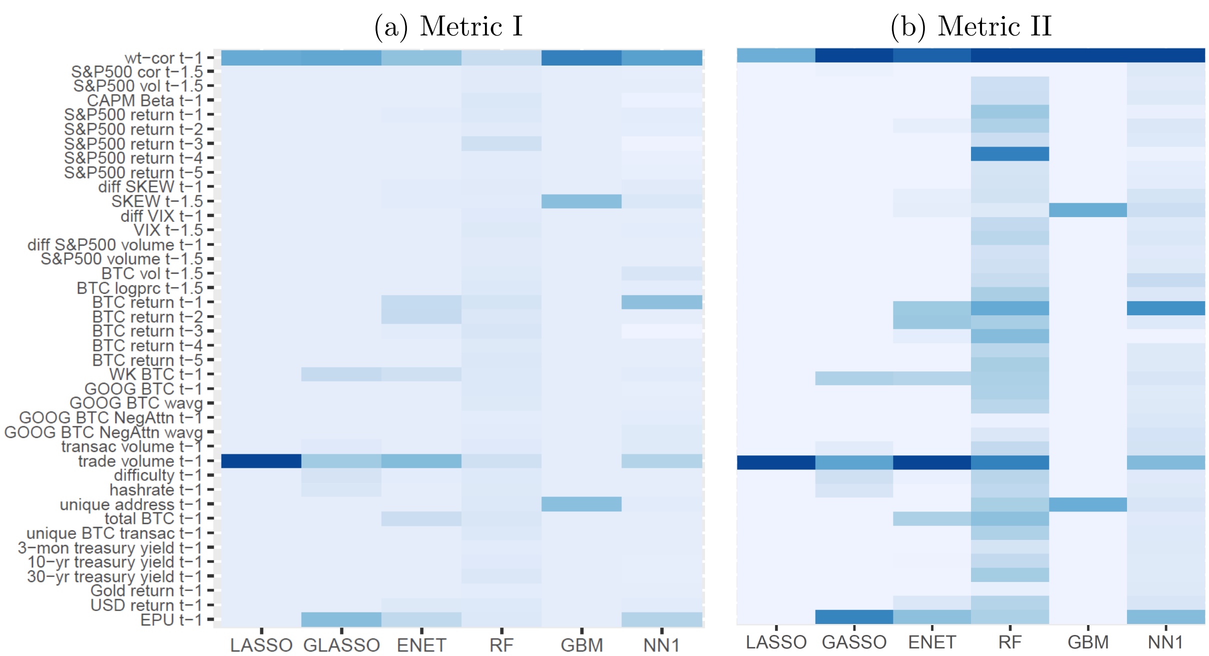


Table A.4: Importance of Predictor Variables

The table shows variable importance metrics (Metric I in Figure A.5), which are reductions in R^2 when a given predictor is set to zero in a trained model (Gu, Kelly, and Xiu 2020), for $\Delta w^{(cor)}_{b,(t-1):t}$ and the second important predictor under different Machine Learning algorithms: LASSO, Group LASSO (GLASSO), Elastic Net (ENET), Random Forest (RF), Gradient Boosting Machine (GBM), and Neural Network with one hidden layer (NN1). The variable importance metric is scaled by the sum of all predictors so that the total sum for each algorithm can be unity. 'MIV' represents the most important variable whereas 'SIV' means the second most important variable. 'VI score' is a variable importance score implied by the variable importance metric.

	LASSO	GLASSO	ENET	RF	GBM	NN1
MIV	lagged trading volume	$\Delta w^{(cor)}_{b,(t-1):t}$	lagged trading volume	$\Delta w^{(cor)}_{b,(t-1):t}$	$\Delta w^{(cor)}_{b,(t-1):t}$	$\Delta w^{(cor)}_{b,(t-1):t}$
VI score of MIV	0.67	0.35	0.27	0.08	0.50	0.38
SIV	$\Delta w^{(cor)}_{b,(t-1):t}$	lagged EPU	$\Delta w^{(cor)}_{b,(t-1):t}$	lagged trading volume	lagged market skewness	lagged Bitcoin returns
VI score of SIV	0.33	0.26	0.24	0.06	0.25	0.24